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Abstract

This paper analyzes how trade integration may affect international financial flows in a world with heterogeneous financial development. In the presence of financial frictions and sector-specific minimum investment requirements, the static gains from trade trigger the cross-sector investment reallocation on the extensive margin, which may allow the more financially developed country (North) to offshore low-return production activities and upgrade to high-return activities. This way, trade-driven sectoral upgrading in North becomes a mechanism through which the substantial decline in trade and communication costs and the resulting boom in supply-chain trade may contribute to the global imbalances in the recent decades.

Keywords: financial frictions, global imbalances, minimum investment requirements, sectoral shifts, supply-chain trade
JEL Classification: F11, F41

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The recent wave of globalization has two prominent features. First, emerging economies (especially China and other emerging Asian economies) have witnessed large current account surplus, while advanced economies (notably, the United States) have incurred persistent current account deficits in the past two decades. Accordingly, financial flows have been "uphill" from the poor to the rich countries (Kose et al., 2010; Prasad, Rajan, and Subramanian, 2006). Such global imbalances differ significantly from the previous episodes and are in stark contrast to the predictions of neoclassical theories. Second, due to technological progress and the removal of trade barriers, the costs of transportation, communication, and coordination have declined dramatically since 1990s, which accelerates international fragmentation of production (Baldwin, 2013b; Grossman and Rossi-Hansberg, 2006; Timmer et al., 2014). Nowadays, international production, trade and investments are organized within global value chains, which have substantially transformed the landscape of global production networks and influenced trade policies (Amador and Cabral, 2016; Antràs, 2015; Park, Nayyar, and Low, 2013).

Traditionally, trade and capital flows have been analyzed separately in the literature and economists have put little research effort on their interactions. Recent works suggest that such a separation is not always innocuous (Eaton et al., 2016; Ghironi and Melitz, 2005; Jin, 2012; Ju, Shi, and Wei, 2014). In a critical contribution to this literature, Antras and Caballero (2009) show that if the global imbalances are an equilibrium response to heterogeneous degrees of financial development across the world, deepening trade integration raises the return to capital in less financially developed countries (South) and resolves the global imbalances, while trade protectionism may backfire. Their findings have profound implications to the political debates at the peak of global imbalances in 2007-2008. However, the recent global imbalances have emerged and accelerated in parallel to world-wide trade liberalization, which seems at odds with their predictions.

Instead of focusing on the interest rate responses of South to trade flows, I emphasize the impacts of trade integration on the industrial structure in the more financially developed country (North). If trade-driven sectoral shifts allow North to offshore low-return activities and upgrade to high-return activities, trade integration may further raise the interest rate in North, which amplifies the global imbalances. The intuition is as follows.

Consider a two-sector, overlapping-generation model where firms hire physical capital and labor to produce two goods which are combined in the Cobb-Douglas aggregator for consumption and investment. The two sectors are symmetric, except that individual investment in a particular sector is subject to the minimum investment requirements (hereafter, MIR). For simplicity, the sector with the MIR is called *sector 1*, while the other sector is called *sector 0*. In the absence of financial frictions, domestic savings are allocated efficiently across sectors. Accordingly, the output price is equalized in the two sectors and so is the sectoral rate of return.

In the presence of financial frictions and the heterogeneity in individual wealth, the agents with sufficiently high net wealth can meet the MIR and invest in sector 1, and they are called

entrepreneurs; other agents have to invest in sector 0 or lend to the credit market, and they are called households. If financial frictions are sufficiently severe, the mass of entrepreneurs is inefficiently low and so are the investment and the output in sector 1. Thus, the rate of return and the output price are higher in sector 1 than in sector 0. The lower the level of financial development, the tighter the borrowing constraints, the smaller the mass of entrepreneurs, the larger the distortions on sectoral investment and output, the larger the sectoral rate-of-return and price differentials, the lower the income per capita. Here, the *extensive margin of investment*[1] is a key channel through which financial frictions and the sector-specific MIR distort the aggregate investment efficiency.

Consider a small open economy (North) which is more financially developed than the rest of the world (South). At the autarkic steady state, the rate of return in sector 0 and the income per capital are higher in North than in South. As investing in sector 0 and lending are perfect substitutes, the interest rate is coupled with rate of return in sector 0, which is higher in North than in South. If allowed, financial flows are "uphill" from South to North. In this model, the global imbalances arise as an equilibrium response to heterogeneous financial development across the world. Meanwhile, at the autarkic steady state, the output price in sector 1 (0) is lower (higher) in North than in South, implying that financial development is a determinant of comparative advantage for trade.

Staring from the autarkic steady state, free trade in the two goods induces North to specialize towards sector 1 and the static gains raise its national income, which affects the sectoral investment in two ways. First, the higher national income allows each agents to invest more, which raises the sectoral level investments in equal proportions along **the intensive margin**. The decreasing MRK (marginal revenue of capital) effect then **dampens** the rise in the national income in the next period. Second, the higher national income allows more agents to overcome the MIR, which reallocates domestic investment towards sector 1 along **the extensive margin** and reinforces North's comparative advantage. In the next period, the enhanced specialization **amplifies** the rise in the national income. As long as the investment reallocation effect dominates the decreasing MRK effect, North's national income rises over time until the mass of entrepreneurs becomes so large that their total borrowing capacity exceeds the entire household saving. In that case, North offshores sector 0 and specializes fully in sector 1. The investment reallocation effect is then mute and the decreasing MRK effect alone brings North to a new steady state. This way, the *extensive margin of investment* is the key channel through which trade may induce North to offshore the low-return sector, which fundamentally changes the way the interest rate is determined, as explained below.

When North specializes **partially** towards sector 1, trade reverses the cross-country rate-of-return pattern in sector 0, as shown by Antras and Caballero (2009). Coupled with the rate of

[1] In each sector, the total investment depends on the investment size of individual investors (the intensive margin) as well as the mass of investors (the extensive margin).

return in sector 0, the interest rate is lower in North than in South, a result Antras and Caballero (2009) calls "**the interest rate reversal**". If allowed, financial flows are "downhill" from North to South. In this case, *trade resolves the global imbalances*. When North eventually offshores sector 0 and specializes **fully** in sector 1, the interest rate is **decoupled** from the rate of return in sector 0 and the mechanism of Antras and Caballero (2009) ceases to work. Coupled with the rate of return in sector 1, the interest rate in North is hence higher than at the autarkic steady state, a result I call "**the interest rate re-reversal**". In this case, *trade amplifies the global imbalances*. To sum up, whether trade resolves or amplifies the global imbalances depends on how far it affects the sectoral composition in the more financially developed country.

In the two-sector setting, the interest rate re-reversal occurs when North fully specializes in the high-return sector, which seems too extreme. In fact, the core mechanism can be easily embedded into a multi-sector setting where sectors are ranked in terms of the MIR. In the presence of sufficiently tight borrowing constraints, the higher the sector-specific MIR, the more severe the sectoral underinvestment problems, the higher the sectoral rate of return. If cross-sector heterogeneity in the MIR and cross-country heterogeneity in financial development are sufficiently large, trade may allow North to **sequentially** offshore the low-MIR, low-return sectors and upgrade to the high-MIR, high-return sectors over time. Once North offshores the lowest-return sector, the interest rate is decoupled from the rate of return in that sector and coupled with the one in the next lowest-return sector. This way, the interest rate in North becomes an inverse sawtooth wave so that *North witnesses the recurrent pattern of the interest rate reversal and re-reversal over time.*

In the following, I use this mechanism to explain the implications of the recent boom in supply-chain trade on the global imbalances.

Supply-Chain Trade and the Global Imbalances In manufacturing industries, upstream activities (such as R&D, product design, or the manufacturing of key parts and components) and downstream activities (such as marketing, brand building, and customer service) constitute a large share of value-added, while the intermediate production stages (such as component fabrication and final assembly) account for a small value-added share (Kimura, 2003). Stan Shih, the founder of Acer, introduced the smile curve to feature such an "U-shaped" value-added pattern along the production chain (Baldwin, Ito, and Sato, 2014; Ye, Meng, and Wei, 2015). Since the 1970s, fabrication and assembly activities account for an increasingly lower value-added share, in OECD countries, compared with upstream and downstream activities (Baldwin, 2013a; Baldwin and Lopez-Gonzalez, 2015; Gereffi, 1999; Koopman, Wang, and Wei, 2014; OECD, 2013). These facts can be replicated in my model as follows.

Fabrication and assembly are involved intensively with standardized, routine tasks that require mainly the input of tangible investment; upstream and downstream activities are mostly involved with knowledge-intensive, non-routine tasks that require heavily the input of intangible investment. Compared with tangible investment, intangible investment has a higher MIR

(particularly in terms of human capital) and faces more severe financial frictions. In my model, "sectors" can be interpreted broadly as production stages or tasks, while investment can be tangible or intangible. One can map fabrication and assembly activities into "sector 0" and map upstream and downstream activities into "sector 1".[2] My model predicts that the output price and the investment return are higher in upstream and downstream activities than in fabrication and assembly. Thus, one can use the smile curve to characterize the rate-of-return pattern across stages.[3]

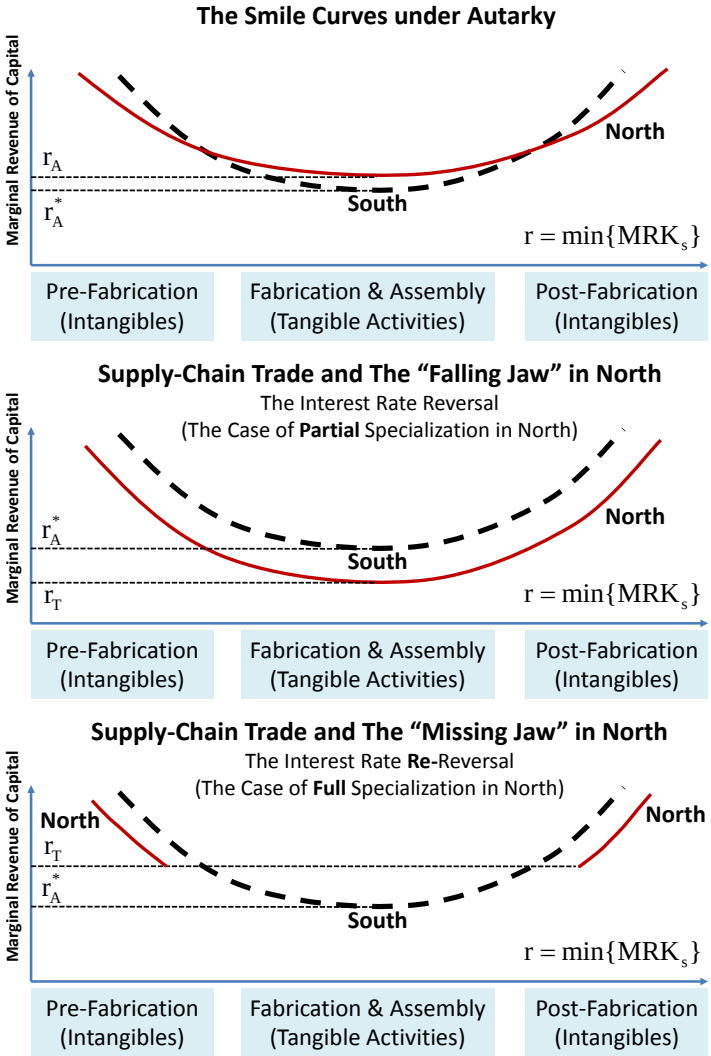

Figure 1: The "Falling vs. Missing Jaw" of the Smile Curve in North

In my model, advanced economies (North) are more developed than emerging economies (South) in financing intangible investment (Corrado et al., 2013). Thus, the sectoral rate-of-return differential is smaller and the smile curve is flatter in North than in South. See the

[2]One can disaggregate upstream and downstream activities into multiple stages, according to their respective MIR. As shown in the multi-sector setting, high-MIR stages have higher returns.

[3]The original smile curve shows the value-added pattern along the production chain. Since the production stages/tasks that account for the high value-added shares usually have the high MRK, I use the smile curve to show the MRK pattern along the value chain.

upper panel of figure 1 where variables in North (South) are denoted without (with) the asterisk superscript. The interest rate is coupled with the MRK in the *active, lowest-return* production stages/tasks, $r = \min\{MRK_s \| K_s > 0\}$, where subscript s is the production stage index. At the autarkic steady state, the interest rate is higher in North than in South, $r_A > r_A^*$. Thus, cross-country heterogeneity in financial development helps explain the "uphill" capital flows and the global imbalances. Besides, North (South) has a comparative advantage in upstream and downstream (fabrication and assembly) activities.

Consider North as a small open economy. In the presence of high trade costs, the incentive of offshoring is small and a marginal decline in trade costs only allows North to offshore *part* of fabrication and assembly, leading to a **"falling jaw"** of its smile curve, as shown in the middle panel of figure 1. It is consistent with the dynamics of value-added shares in OECD countries since the 1970s. As the interest rate in North is still coupled with the MRK in fabrication and assembly, the *interest rate reversal* happens, i.e., $r_T < r_A^* < r_A$.

Since the 1990s, technological progress and world-wide economic liberalization have substantially reduced the costs of transportation, communication, and coordination, leading to the boom of supply-chain trade and the expansion of global production networks. Vertically and horizontally linked production stages/tasks have been increasingly conducted in different countries. In my model, a substantial decline in trade costs allows North to offshore *all* fabrication/assembly and specialize fully in upstream/downstream activities. Then, the smile curve in North witnesses a **"missing jaw"** so that the interest rate is coupled with the MRK in upstream and downstream activities. As shown the lower panel of figure 1, supply-chain trade leads to the *interest rate re-reversal*, which amplifies the initial interest rate differential, $r_T > r_A > r_A^*$.

To sum up, if supply-chain trade allows North to sequentially offshore the low-return activities and upgrade along the value chain, the cross-country interest rate differential can be maintained or even amplified. This way, **trade-driven sectoral upgrading** can be a critical mechanism through which the recent boom in supply-chain trade may contribute to the global imbalances. It complements the findings of Antras and Caballero (2009). Undoubtedly, various factors, e.g., globalization, technology progress, industrial policies, and etc., may contribute to the sectoral upgrading of advanced economies along the value chain. No matter what the causes are, as long as they offshore the low-return activities and climb up the value chain, the structural shifts in industrial composition may fundamentally change the way the interest rate is determined and the patterns of international financial flows. The core message of this paper is that the non-monotonic impacts of economic integration on industrial composition can have profound implications to trade and capital flows.

Related Literature This paper is closely related to the recent literature on explaining the global imbalances as an equilibrium response to heterogeneous financial development across countries (Caballero, Farhi, and Gourinchas, 2008; Gourinchas and Rey, 2014; Ju and Wei, 2010; Mendoza, Quadrini, and Rios-Rull, 2009; von Hagen and Zhang, 2014; Zhang, 2017).

Building upon this literature, Antras and Caballero (2009) show that deepening trade integration leads to the interest rate reversal, which resolves the global imbalances. They focus on the interest rate response to trade flows in South where both constrained and unconstrained sectors are always active. However, they do not explore explicitly whether and under what conditions trade flows may allow North to fully offshore the low-return sector.[4] By introducing sector-specific MIR and wealth heterogeneity into the basic model of Antras and Caballero (2009), I endogenize the extensive margin of investment. It is exactly through this channel that the static gains from trade triggers cross-sector investment reallocation, leading to the possibility of North fully offshoring the low-return activities and the interest rate re-reversal.

Grossman and Rossi-Hansberg (2006) and Baldwin (2013a,b) show that the recent wave of offshoring and supply-chain trade has changed the composition of world trade and transformed the industrial structures in advanced and emerging economies. By proposing a mechanism through which supply-chain trade affects the industrial structures in advanced economies, I investigate its implications to international financial flows.

Kletzer and Bardhan (1987) were the first to show that better access to capital becomes a source of comparative advantage. It was then followed by a strand of theoretical literature on financial development and international trade (Antras and Caballero, 2009; Beck, 2002; Chesnokova, 2007; Ju and Wei, 2005).[5] Matsuyama (2005) introduces sector-specific borrowing constraints in a static model and shows that trade allows the rich (poor) country to fully specialize in the sector with tighter (looser) borrowing constraints. Wynne (2005) argues that a country's wealth can be a determinant of comparative advantage when access to credit differs across sectors, i.e., wealthier nations exhibit a comprehensive advantage towards goods produced in sectors facing more severe financial frictions. Ju and Wei (2011) point out that, in the countries with low-quality institutions, the quality of financial system is an independent source of comparative advantage. Building upon this literature, I analyze the joint determination of trade and capital flows.

Jin (2012) integrates factor-proportions-based trade and financial flows in an OLG model and shows that capital tends to flow to countries that become more specialized in capital-intensive industries. Ju, Shi, and Wei (2014) incorporate two tradeable sectors with different factor intensity in a small-open-economy setting and show that the current account adjustment

[4]See subsection 4.1 for a detailed discussion.

[5]Recently, a booming literature has documented the extensive empirical evidence on the relationship between financial development and trade patterns. Financially developed countries export more in sectors that require more external finance and in sectors with fewer tangible assets (Beck, 2003; Hur, Raj, and Riyanto, 2006; Svaleryd and Vlachos, 2005). Manova (2008) shows that equity market liberalization increases exports disproportionately more in sectors that require more external funds or employ fewer collateralizable assets. Manova (2013) further decompose the trade effect of weak financial markets and show that financially developed countries serve more destination markets and export more products, in more financially vulnerable sectors. Chor and Manova (2012) analyze the collapse of international trade flows during the global financial crisis and show that credit conditions were an important channel through which the financial crisis affected trade volumes.

to exogenous shocks depends on factor market flexibility. Instead of introducing sector-specific financial frictions or factor intensity, I focus on a real friction, i.e., sector-specific MIR.

In the literature, the MIR is used to capture investment indivisibility at the individual level, an important feature of business ideas, physical and human capital (Aghion and Bolton, 1997; Banerjee and Moll, 2010; Banerjee and Newman, 1993; Galor and Zeira, 1993; Piketty, 1997; Zhang, 2017). Erosa and Hidalgo-Cabrillana (2008), Barseghyan and DiCecio (2011), Buera, Kaboski, and Shin (2011), Manova (2013), and Midrigan and Xu (2014) introduce the fixed cost or the entry cost at the firm level and show that the individual investment is above a minimum scale in equilibrium. Assuming the MIR rather than the fixed cost allows me to characterize analytically the dynamic properties in the entire parameter space.

I also revisit the factor price equalization (hereafter FPE) theorem in the presence of financial and real frictions. In Antras and Caballero (2009), although trade alone does not lead to the FPE, allowing both trade and capital flows can do. In my model, if trade has already induced North to offshore the lower-return sector, adding capital mobility does not achieve the FPE, which is opposite to the finding of Antras and Caballero (2009). In this case, *the timing and the sequence of economic integration* are critical for the FPE.

The rest of the paper is structured as follows. Section 1 sets up the model and section 2 analyzes the autarkic equilibrium. Section 3 discusses the impacts of trade on national income and the interest rate, given the world economy initially at the autarkic steady state. Section 4 explores the channels for the interest rate re-reversal, discusses the implications of trade costs and initial conditions, and extends the two-sector setting into a three-sector setting. Section 5 makes some final remarks. The appendix covers technical proofs and related materials.

1 The Model Setting

The world consists of two countries, North and South, which are inherently identical except for the population size and the level of financial development, as specified later. In this section, I first describe the economic setting of North and then use the asterisk superscript to denote the variables and the parameters in South.

In North, a continuum of agents are born every period and live for two periods, young and old. In each generation, the population size is constant at one and agents are indexed by $j \in [0, 1]$. Agent j is endowed with $l_j = (1-\theta)\epsilon_j$ units of labor when young, where $\epsilon_j \in (1, \infty)$ follows the Pareto distribution, $G(\epsilon_j) = 1 - \epsilon_j^{-\frac{1}{\theta}}$ and $\theta \in (0, 1)$.[6] Agents only consume when

[6]The inverse of θ is the tail index of the Pareto distribution. Pareto distribution is widely used in the literature to feature the income and wealth distribution (Atkinson, Piketty, and Saez, 2011; Gabaix, 2009; Jones, 2015). The top tail of income distribution is very well approximated by a Pareto distribution (Kuznets and Jenks, 1953; Piketty and Saez, 2003).

old and they supply the labor endowment inelastically to the market when young. Every period, the aggregate labor supply is constant at $L = \int_1^\infty l_j dG(\epsilon_j) = 1$.

North has two sectors, indexed by $s \in \{0, 1\}$. In period t, $K_{s,t}$ units of physical capital and $L_{s,t}$ units of labor are hired in sector s to produce $Y_{s,t}$ units of good s. Physical capital fully depreciates after use. Sectoral outputs are tradable, while labor and physical capital are not. $V_{0,t}$ units of good 0 and $V_{1,t}$ units of good 1 are combined to produce Y_t units of final goods which are used for consumption and investment.[7] The markets for goods and productive factors are competitive and final goods serve as the numeraire[8]. There is no uncertainty in the model economy. Let w_t denote the wage rate. Let $p_{s,t}$ and $q_{s,t}$ denote respectively the output price and the rental price of capital in sector s.

$$Y_{s,t} = \left(\frac{K_{s,t}}{\alpha}\right)^{\alpha} \left(\frac{L_{s,t}}{1-\alpha}\right)^{1-\alpha}, q_{s,t}K_{s,t} = \alpha p_{s,t}Y_{s,t}, w_tL_{s,t} = (1-\alpha)p_{s,t}Y_{s,t}, \tag{1}$$

$$Y_t = \left(\frac{V_{1,t}}{\eta}\right)^{\eta} \left(\frac{V_{0,t}}{1-\eta}\right)^{1-\eta}, \ p_{1,t}V_{1,t} = \eta Y_t, \ p_{0,t}V_{0,t} = (1-\eta)Y_t. \tag{2}$$

where $\alpha, \eta \in (0, 1)$. In order to focus on the interest rate response to trade, I exclude international capital flows so that domestic investment is funded by domestic saving and national income is equal to domestic output. Combine (1)-(2) to get $w_t = (1-\alpha)\frac{Y_t}{L}$. Thus, the wage rate can serve as a proxy of national income. Later, I use the law of motion for wage to analyze the model dynamics and the steady-state properties.

As labor is perfectly mobile across sectors and the labor market is frictionless, the wage rate equalizes the marginal revenue of labor (hereafter MRL) in both sectors. As shown below, physical capital is sector-specific and domestic investment is subject to frictions. Thus, the marginal revenue of capital (hereafter, MRK) may differ across sectors, leading to the sectoral rental-price-of-capital wedge.

In period t, young agents invest their entire labor income w_tL in the two sectors, which yields in period $t+1$ $K_{s,t+1}$ units of physical capital in sector s. Let δ_t and ζ_{t+1} denote respectively the fractions of domestic saving (w_tL) and labor endowment (L) allocated for the production of good 1 in period $t+1$.

$$K_{1,t+1} = \delta_t w_t L, \qquad L_{1,t+1} = \zeta_{t+1}L, \tag{3}$$

$$K_{0,t+1} = (1-\delta_t)w_t L, \qquad L_{0,t+1} = (1-\zeta_{t+1})L. \tag{4}$$

[7]Under autarky, domestic absorption is equal to domestic output at the sectoral level, $V_{s,t} = Y_{s,t}$, while it is not the case with trade flows, $V_{s,t} \neq Y_{s,t}$.

[8]In Antras and Caballero (2009), capital and labor are used to produce two final goods which are combined in the Cobb-Douglas aggregator for consumption and investment, while one final good is chosen as the numeraire. In Ju and Wei (2011), the two final goods are combined to produce a composite good in a Cobb-Douglas fashion, while the composite good acts as the numeraire and is used for consumption and investment. My results do not depend on the choice of numeraire.

Let $\chi_t \equiv \frac{p_{0,t}}{p_{1,t}}$ and $\mu_t \equiv \frac{q_{0,t}}{q_{1,t}}$ denote respectively the sectoral output-price ratio and the sectoral rental-price-of-capital ratio. Combine equations (1) and (3)-(4),

$$p_{s,t} = (q_{s,t})^{\alpha} w_t^{1-\alpha}, \quad \Rightarrow \quad \chi_t = \mu_t^{\alpha}, \tag{5}$$

$$\zeta_{t+1} = \frac{\delta_t}{(1-\delta_t)\chi_{t+1}^{\frac{1}{\alpha}} + \delta_t} = \frac{\delta_t}{(1-\delta_t)\mu_{t+1} + \delta_t}. \tag{6}$$

Given aggregate investment $w_t L$ and labor input L, equation (6) implicitly describes the production possibility frontier (PPF, hereafter) for North in period $t+1$,[9] reflecting the sectoral output supply. Equation (2) essentially reflects the isoquant, featuring the sectoral output demand. In the next sections, I will combine the isoquant and the PPF to show the equilibrium allocations under autarky and under free trade, respectively. In the following, I first derive explicitly the PPF, given the investment distortions.

Agent j born in period t can save its labor income $n_{j,t} = w_t l_j$ in three ways: investing in the two sectors and lending to the credit market. By investing $k_{j,0,t+1} > 0$ units of final goods in period t, the agent gets in period $t+1$ $k_{j,0,t+1}$ units of physical capital for sector 0 and the rate of return to its investment is $q_{0,t+1}$. It gets the same one-to-one investment outcome in sector 1, if its investment size meets the MIR, $k_{j,1,t+1} \geq \mathfrak{m} > 0$; otherwise, its investment in sector 1 yields zero outcome.

Except for the MIR, the two sectors are symmetric in terms of the investment and production technologies. Let $\mathbb{Y}_{t+1} \equiv \left(\frac{w_t L}{\alpha}\right)^{\alpha} \left(\frac{L}{1-\alpha}\right)^{1-\alpha}$ denote a sector's maximum possible output if domestic saving and labor are fully allocated in that sector. If credit markets were perfect so that everyone could meet the MIR, investment in the two sectors would be perfect substitutes and the sectoral rates of return would equalize under autarky, $q_{0,t+1} = q_{1,t+1}$ and $\mu_{t+1} = 1$. In this case, for the output of $Y_{1,t+1}$, productive factors would be allocated in sector 1 in equal proportions, $\zeta_{t+1} = \delta_t = \frac{Y_{1,t+1}}{\mathbb{Y}_{t+1}}$, according to equations (6), (3), and (1). Thus, the PPF is linear and the marginal rate of transformation (MRT, hereafter) is constant at unity,

$$Y_{0,t+1} + Y_{1,t+1} = \mathbb{Y}_{t+1}, \quad MRT_{0,1} \equiv -\frac{\partial Y_{1,t+1}}{\partial Y_{0,t+1}} = \chi_{t+1} = 1. \tag{7}$$

However, due to limited commitment, agent j can borrow only up to a fraction λ of its investment in sector 1 $k_{j,1,t+1}$ and has to use its own funds $n_{j,t}$ to cover the financing gap,

$$k_{j,1,t+1} - n_{j,t} \leq \lambda k_{j,1,t+1}, \tag{8}$$

where $\lambda \in [0,1)$ measures the level of financial development.[10] If the borrowing constraints are sufficiently tight, the mass of agents who can meet the MIR is inefficiently low and so is the total investment in sector 1. In this case, financial frictions and sector-specific MIR jointly

[9] See the proof of Lemma 1 for derivation.

[10] Antras and Caballero (2009) give some micro-foundations for this borrowing constraint. The credit markets would be perfect and the borrowing constraints would be slack if $\lambda = 1$.

distort the sectoral investment, leading to the sectoral return wedge, $q_{1,t+1} > q_{0,t+1}$. According to equation (6), $\mu_{t+1} < 1$ implies that capital and labor are not allocated in sector 1 in equal proportions, $\zeta_{t+1} > \delta_t$. I derive the PPF for this case as follows.

Since all agents can freely invest in sector 0 and lend to the credit market, these two options are perfect substitutes. Under autarky, $K_{0,t+1} > 0$ implies that the interest rate is coupled with the rate of return in sector 0. As shown in section 3, trade may induce North to offshore sector 0 and specialize fully in sector 1. In that case, the interest rate is decoupled from (coupled with) the rate of return in sector 0 (1).

$$r_t = \begin{cases} q_{0,t+1}, & \text{if } K_{0,t+1} > 0 \text{ or equivalently } \delta_t < 1; \\ q_{1,t+1}, & \text{if } K_{0,t+1} = 0 \text{ or equivalently } \delta_t = 1. \end{cases} \tag{9}$$

Consider first the case where investment frictions lead to the sectoral return wedge, $q_{1,t+1} > q_{0,t+1}$, and sector 0 is still active, $K_{0,t+1} > 0$. In period t, the positive rate-of-return spread induces agent j to invest its entire labor income in sector 1 and borrow to the limit, if it can meet the MIR, $k_{j,1,t+1} = \frac{n_{j,t}}{1-\lambda} \geq \mathfrak{m}$. In period $t+1$, the agent gets the investment return, repays the debt, and consumes the rest. Let $\Omega_{j,t} \equiv \frac{q_{1,t+1}k_{j,1,t+1} - r_t(k_{j,1,t+1} - n_{j,t})}{n_{j,t}}$ denote agent j's equity rate, i..e, the rate of return to its own funds. The binding borrowing constraints imply that the equity rate is common for all agents who meet the MIR and the agent's investment size in sector 1 is linear in its net wealth.

$$\Omega_{j,t} = \Omega_t \equiv q_{1,t+1} + (q_{1,t+1} - r_t)(\frac{1}{1-\lambda} - 1) > q_{1,t+1} > r_t, \tag{10}$$

$$k_{j,1,t+1} = \frac{n_{j,t}}{1-\lambda} = \frac{w_t}{1-\lambda}(1-\theta)\epsilon_j \geq \mathfrak{m}, \;\; \Rightarrow \;\; \epsilon_j \geq \underline{\epsilon}_t \equiv \frac{1-\lambda}{w_t}\frac{\mathfrak{m}}{1-\theta}, \tag{11}$$

where $\underline{\epsilon}_t$ is a cutoff value. Agents with $\epsilon_j \geq \underline{\epsilon}_t$ can meet the MIR and they are called *entrepreneurs*, with the mass of $\tau_t = \underline{\epsilon}_t^{-\frac{1}{\theta}}$. When young, they invest in sector 1 with their entire labor income $n_{j,t}$ and borrow to the limit $k_{j,1,t+1} - n_{j,t} = \frac{\lambda n_{j,t}}{1-\lambda}$; when old, they consume $n_{j,t}\Omega_t$. In period t, the fraction of domestic saving allocated in sector 1 is

$$\delta_t = \frac{\int_{\underline{\epsilon}_t}^{\infty} k_{j,1,t+1} dG(\epsilon_j)}{w_t L} = \frac{\underline{\epsilon}_t^{-\frac{1-\theta}{\theta}}}{1-\lambda} = \left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}}, \text{ where } \bar{w} \equiv \frac{\mathfrak{m}(1-\lambda)^{\frac{1}{1-\theta}}}{1-\theta}. \tag{12}$$

Agents with $\epsilon_j \in [1, \underline{\epsilon}_t)$ cannot meet the MIR and are called *households*. When young, they invest $k_{j,0,t+1}$ in sector 0 and lend out $n_{j,t} - k_{j,0,t+1}$; when old, they consume $n_{j,t}r_t$,

$$k_{j,0,t+1} + (n_{j,t} - k_{j,0,t+1}) = n_{j,t} = w_t l_j. \tag{13}$$

The markets for credit, sector-specific capital, labor, and final goods clear,

$$\int_{\underline{\epsilon}_t}^{\infty} (k_{j,1,t+1} - n_{j,t}) dG(\epsilon_j) = \int_1^{\underline{\epsilon}_t} (n_{j,t} - k_{j,0,t+1}) dG(\epsilon_j), \tag{14}$$

$$K_{1,t} = \delta_{t-1} w_{t-1} L, \;\; K_{0,t} = (1-\delta_{t-1}) w_{t-1} L, \;\; L_{0,t} + L_{1,t} = L, \tag{15}$$

$$\int_{\underline{\epsilon}_t}^{\infty} n_{j,t-1}\Omega_{t-1} dG(\epsilon_j) + \int_1^{\underline{\epsilon}_t} n_{j,t-1} r_{t-1} dG(\epsilon_j) + K_{1,t+1} + K_{0,t+1} = Y_t. \tag{16}$$

Given the binding borrowing constraints in period t, δ_t is predetermined by equation (12) from the perspective of period $t+1$. Thus, the PPF in period $t+1$ is

$$\left[\frac{Y_{0,t+1}}{(1-\delta_t)^{\alpha}}\right]^{\frac{1}{1-\alpha}} + \left(\frac{Y_{1,t+1}}{\delta_t^{\alpha}}\right)^{\frac{1}{1-\alpha}} = \mathbb{Y}_{t+1}^{\frac{1}{1-\alpha}}, MRT_{0,1} \equiv -\frac{\partial Y_{1,t+1}}{\partial Y_{0,t+1}} = \chi_{t+1} < 1. \tag{17}$$

Consider then the case of $q_{1,t+1} = q_{0,t+1}$. Due to the zero spread, the agents who can meet the MIR do not have the incentive to invest their entire labor income in sector 1 or to borrow to the limit. Despite the indeterminacy at the individual level, $\mu_{t+1} = 1$ implies that the PPF in period $t+1$ is specified by equation (7).

Let $\tilde{\delta}_t \equiv \min\{\left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}}, 1\}$ denote the maximum possible share of domestic investment in sector 1 when all entrepreneurs borrow and invest to the limit.

Lemma 1. *Let $\tilde{Y}_{1,t+1} \equiv \tilde{\delta}_t \mathbb{Y}_{t+1}$. If $w_t < \bar{w}$, $\tilde{\delta}_t < 1$ and the PPF in period $t+1$ is piecewise and consists of two parts. For $Y_{1,t+1} > \tilde{Y}_{1,t+1}$, the PPF is concave with $\delta_t = \tilde{\delta}_t$, as specified by equation* (17)*; for $Y_{1,t+1} \leq \tilde{Y}_{1,t+1}$, the PPF is linear with $\delta_t = \frac{Y_{1,t+1}}{\mathbb{Y}_{t+1}} < \tilde{\delta}_t$, as specified by equation* (7)*.*

If $w_t \geq \bar{w}$, $\tilde{\delta}_t = 1$ and the entire PPF is linear, as specified by equation (7)*.*

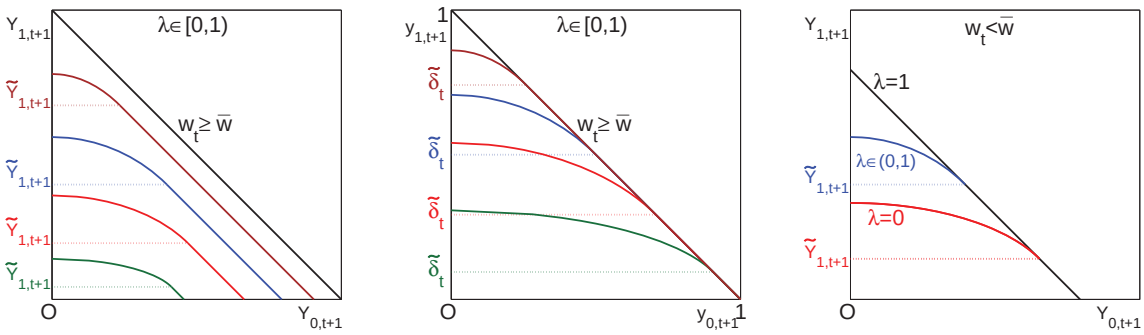

Figure 2: The Production Possibility Frontiers

$\tilde{\delta}_t$ depends positively on the level of financial development (λ) and the level of national income (with w_t as a proxy).

- For the output of $Y_{1,t+1} \leq \tilde{Y}_{1,t+1}$, equation (7) specifies the efficient fraction of investment in sector 1, $\hat{\delta}_t = \frac{Y_{1,t+1}}{\mathbb{Y}_{t+1}}$, which is feasible $\hat{\delta}_t \leq \tilde{\delta}_t = \frac{\tilde{Y}_{1,t+1}}{\mathbb{Y}_{t+1}}$. In equilibrium, the sectoral investment is efficient, $\delta_t = \hat{\delta}_t \leq \tilde{\delta}_t$, and the sectoral rates of return equalize, $\mu_{t+1} = 1$. A marginal rise in $Y_{1,t+1}$ requires the reallocation of both labor and investment from sector 0 to 1 in equal proportions, $\zeta_{t+1} = \delta_t$. As a result, $Y_{0,t+1}$ declines by the same amount, implying a linear PPF, as specified by equation (7).

- For the output of $Y_{1,t+1} > \tilde{Y}_{1,t+1}$, the efficient fraction of investment in sector 1 is not feasible, $\hat{\delta}_t = \frac{Y_{1,t+1}}{\mathbb{Y}_{t+1}} > \tilde{\delta}_t = \frac{\tilde{Y}_{1,t+1}}{\mathbb{Y}_{t+1}}$. In equilibrium, the sectoral investment in sector 1 is inefficiently low, $\delta_t = \tilde{\delta}_t < \hat{\delta}_t$, leading to the sectoral return wedge, $q_{1,t+1} > q_{0,t+1}$. Given

a predetermined δ_t, a marginal rise in $Y_{1,t+1}$ requires the labor reallocation from sector 0 to 1 more than proportionally. As a result, $Y_{0,t+1}$ falls by a larger amount, implying a concave PPF, as specified by equation (17). If labor is fully allocated in sector 1, the maximum output in sector 1 is $Y_{1,t+1} = \tilde{\delta}_t^{\alpha} \mathbb{Y}_{t+1} < \mathbb{Y}_{t+1}$.

As shown in the left panel of figure 2, given $w_t < \bar{w}$, a rise in w_t affects the PPF in two ways. First, the higher the w_t, the higher the national income, the higher the domestic saving and investment, the higher the $\mathbb{Y}_{t+1}$. I call it the *investment scale effect*, which shifts the PPF away from the origin. Second, the higher the w_t, the larger the mass of entrepreneurs, the higher the $\tilde{\delta}_t$. I call it the *investment composition effect*, which extends the linear fraction of the PPF.

We can explicitly highlight the investment composition effect by normalizing sectoral outputs in equations (7) and (17) by $\mathbb{Y}_{t+1}$,

$$y_{0,t+1} + y_{1,t+1} = 1, \text{and} \left[\frac{y_{0,t+1}}{(1-\tilde{\delta}_t)^{\alpha}} \right]^{\frac{1}{1-\alpha}} + \left(\frac{y_{1,t+1}}{\tilde{\delta}_t^{\alpha}} \right)^{\frac{1}{1-\alpha}} = 1, \text{where } y_{s,t+1} \equiv \frac{Y_{s,t+1}}{\mathbb{Y}_{t+1}}.$$

As shown in the middle panel of figure 2, given $w_t < \bar{w}$, a rise in w_t raises $\tilde{\delta}_t$ so that the linear part accounts for a larger fraction in the normalized PPF.

For $w_t \geq \bar{w}$, the mass of entrepreneurs is so high that domestic saving can be fully invested in sector 1, $\tilde{\delta}_t = 1$. For the output of $Y_{1,t+1} \leq \mathbb{Y}_{t+1}$, the efficient fraction of investment in sector 1 is always feasible, $\hat{\delta}_t = \frac{Y_{1,t+1}}{\mathbb{Y}_{t+1}} \leq \tilde{\delta}_t = 1$. Thus, the PPF becomes linear, as shown by the diagonal lines in the left and middle panels.

The right panel of figure 2 shows that, given $w_t < \bar{w}$, the higher the λ, the larger the mass of entrepreneurs, the higher the $\tilde{\delta}_t$ and $\tilde{Y}_{1,t+1}$, the larger the linear part of the PPF. For $\lambda = 1$, $\tilde{\delta}_t = 1$ and the entire PPF becomes linear.

Under autarky, the sectoral output markets clear domestically,

$$V_{s,t} = Y_{s,t}. \tag{18}$$

Definition 1. *Under autarky, a market equilibrium is a set of choices of agents* $\{n_{j,t}, k_{j,s,t+1}\}$ *and aggregate variables* $\{Y_t, Y_{s,t}, K_{s,t}, L_{s,t}, V_{s,t}, \chi_t, \mu_t, w_t, r_t, \Omega_t, \underline{\epsilon}_t\}$*, satisfying equations* (1)-(2)*,* (9)-(11)*,* (13)-(15)*, and* (18)*, where* $s \in \{0, 1\}$.[11]

Free trade aligns the sectoral price ratio to the world level and trade is balanced,

$$\chi_t = \chi_t^*, \qquad \chi_t^*(V_{0,t} - Y_{0,t}) + (V_{1,t} - Y_{1,t}) = 0. \tag{19}$$

Definition 2. *Under free trade, a market equilibrium is a set of choices of agents* $\{n_{j,t}, k_{j,s,t+1}\}$ *and aggregate variables* $\{Y_t, Y_{s,t}, K_{s,t}, L_{s,t}, V_{s,t}, \mu_t, w_t, r_t, \Omega_t, \underline{\epsilon}_t\}$*, satisfying equations* (1)-(2)*,* (9)-(11)*, and* (13)-(15)*, where* $s \in \{0, 1\}$*, while the sectoral price ratio* χ_t *is determined at the world level by equation* (19)*.*

[11] According to the Walras' law, if the markets for labor, credit, sector-specific physical capital, and sectoral outputs clear, the final good market must clear. In this sense, equation (16) is redundant.

As there is no international mobility of productive factors, domestic investment is financed by domestic saving in period t, $\sum_{v=0}^{1} K_{v,t+1} = w_t L$. According to equations (1)-(2), the aggregate investment return in period $t+1$ is $\sum_{v=0}^{1} q_{v,t+1} K_{v,t+1} = \rho w_{t+1} L$, where $\rho \equiv \frac{\alpha}{1-\alpha}$. Define the social rate of return as

$$\Upsilon_t \equiv \frac{\sum_{v=0}^{1} q_{v,t+1} K_{v,t+1}}{\sum_{v=0}^{1} K_{v,t+1}} = (1-\delta_t) q_{0,t+1} + \delta_t q_{1,t+1} = \rho \frac{w_{t+1}}{w_t}. \tag{20}$$

2 The Autarkic Equilibrium

Due to the frictionless labor market and cross-sector labor mobility, the sectoral allocation of labor is efficient under autarky and equal to the sectoral share in the production of final goods, $\zeta_{t+1} = \eta$. It would also apply to the sectoral allocation of physical capital $\delta_t = \eta$, if the sectoral investments were efficient, i.e., $\mu_{t+1} = 1$.

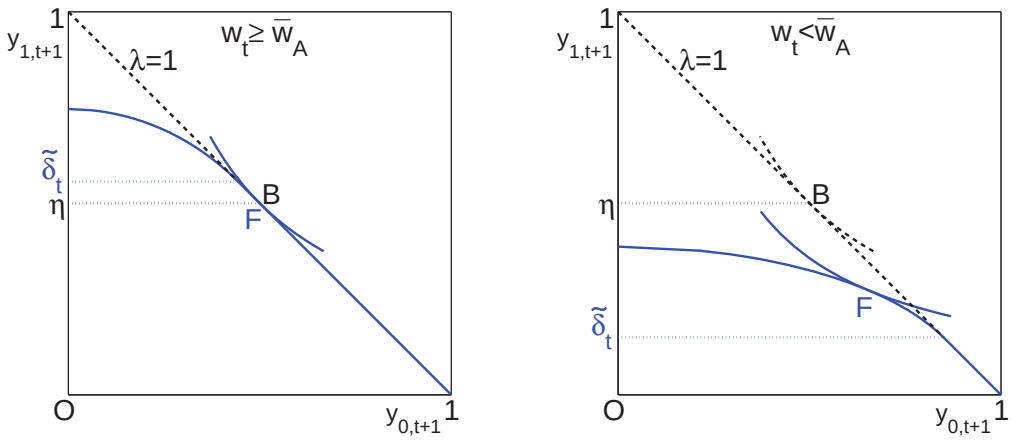

Figure 3: Equilibrium Allocations under Autarky

Let us start with the benchmark case of perfect credit markets, i.e., $\lambda = 1$. In the absence of financial frictions, sectoral investment is efficient and sectoral prices equalize, $\chi_{t+1} = \mu_{t+1} = 1$. The autarkic equilibrium is represented by the tangent point of the isoquant and the PPF. In figure 3, the dashed, diagonal line shows the normalized PPF and point **B** represents the autarkic equilibrium in the **b**enchmark setting. Given the Cobb-Douglas production functions at the sectoral and at the aggregate levels, capital has the decreasing MRK so that the law of motion for wage is concave,

$$w_{t+1} = \left(\frac{w_t}{\rho}\right)^{\alpha}, \frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} < 1, \text{where } \rho \equiv \frac{\alpha}{1-\alpha}. \tag{21}$$

The decreasing MRK is a convergence force that drives North to a unique steady state with the wage rate $w_B = \rho^{-\rho}$. Subscript **B** refers to the ***b***enchmark case.[12] The smaller the α, the stronger the decreasing MRK effect, the faster the convergence.

[12]The dashed curves in figure 5 show the law of motion for wage under autarky and point **B** represents the steady state in the benchmark case.

Consider the case of financial frictions, $\lambda \in [0, 1)$. Let $\bar{w}_A \equiv \eta^{\frac{\theta}{1-\theta}} \bar{w} < \bar{w}$. If $w_t \geq \bar{w}_A$, the efficient investment allocation is feasible, $\eta \leq \tilde{\delta}_t$. In equilibrium, $\delta_t = \zeta_{t+1} = \eta$ and $\chi_{t+1} = \mu_{t+1} = 1$ hold. In the left panel of figure 3, the solid, concave curve shows the normalized PPF, while point **F** shows the autarkic equilibrium with financial **f**rictions and coincides with point B. In this case, the income dynamics are also characterized by equation (21).

If $w_t < \bar{w}_A$, the efficient investment allocation is infeasible, $\eta > \tilde{\delta}_t$ and the under-investment in sector 1, $\delta_t = \tilde{\delta}_t < \zeta_{t+1} = \eta$, leads to the sectoral return wedge, $\mu_{t+1} < 1$. Given w_t, the income dynamics are featured by $\{\delta_t, \mu_{t+1}, w_{t+1}\}$ satisfying (12), (22)-(23),[13]

$$w_{t+1} = \left(\frac{w_t}{\rho}\Gamma_t\right)^{\alpha}, \text{ where } \Gamma_t \equiv \frac{\mu_{t+1}^{\eta}}{1-\eta(1-\mu_{t+1})} < 1, \text{ and } \frac{\partial \Gamma_t}{\partial \mu_{t+1}} > 0, \tag{22}$$

$$\mu_{t+1} = \frac{\frac{1}{\eta} - 1}{\frac{1}{\delta_t} - 1}, \quad \Rightarrow \quad \frac{\partial \mu_{t+1}}{\partial w_t} = \frac{\partial \mu_{t+1}}{\partial \delta_t}\frac{\partial \delta_t}{\partial w_t} > 0, \tag{23}$$

where Γ_t measures the aggregate investment efficiency. Sectoral investment distortion ($\mu_{t+1} < 1$) creates aggregate inefficiency ($\Gamma_t < 1$).[14] In the right panel of figure 3, the solid, concave curve shows the normalized PPF, while the equilibrium allocation represented by point **F** is on an isoquant lower than in the benchmark case. The distance between the two isoquants reflects the efficiency loss in percentage terms, $(1 - \Gamma_t^{\alpha})$.

A rise in national income raises the wage rate, which affects the sectoral investment in two ways. First, it allows agents to invest more so that sectoral investment rises on the *intensive margin*. The decreasing MRK effect *dampens* the initial income change. Second, it allows more agents to meet the MIR and become entrepreneurs, which shifts domestic investment towards sector 1 on the *extensive margin*. This investment composition effect improves aggregate efficiency and *amplifies* the initial income change.

$$\begin{aligned}\frac{\partial \ln w_{t+1}}{\partial \ln w_t} &= 1 - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} + \underbrace{\frac{\partial \ln w_{t+1}}{\partial \ln \Gamma_t}\frac{\partial \ln \Gamma_t}{\partial \ln \mu_{t+1}}\frac{\partial \ln \mu_{t+1}}{\partial \ln w_t}}_{\text{investment composition effect}} \\ &= \alpha + \alpha\eta\frac{1-\theta}{\theta}(1-\mu_{t+1}).\end{aligned} \tag{24}$$

Let X_A denote the steady-state value of variable X_t under **a**utarky. If there is a steady state with $w_A \in (0, \bar{w}_A)$, the slope of the law of motion for wage at this steady state is $\frac{\partial w_{t+1}}{\partial w_t}|_{w_A} = \alpha + \alpha\eta\frac{1-\theta}{\theta}(1-\mu_A)$. With $\mu_A < 1$, the investment composition effect is active. The higher the θ, the more dispersed the wealth distribution, the less sensitive the mass of entrepreneurs to income changes, the weaker the investment composition effect; the lower the α, the stronger the decreasing MRK effect and the weaker the investment composition effect.

Proposition 1. *Let* $\underline{\theta} \equiv \frac{\alpha}{\alpha + \frac{1-\alpha}{\eta}} < \alpha$, $Z \equiv \frac{\frac{1}{\rho}}{\left(\frac{\mathtt{m}}{1-\theta}\eta^{\frac{\theta}{1-\theta}}\right)^{\frac{1}{\rho}}}$, *and* $\tilde{\lambda}_A \equiv 1 - Z^{\rho(1-\theta)}$.

[13] See the proof of proposition 1 in appendix B for derivation.

[14] In the benchmark case, $\mu_{t+1} = 1$ and the aggregate efficiency index is constant at $\Gamma_t = 1$.

For $\theta \in [\underline{\theta}, 1)$ and $\lambda \in [0, \tilde{\lambda}_A)$, there is a unique, stable steady state under autarky where the borrowing constraints are binding and sectoral investment is inefficient.

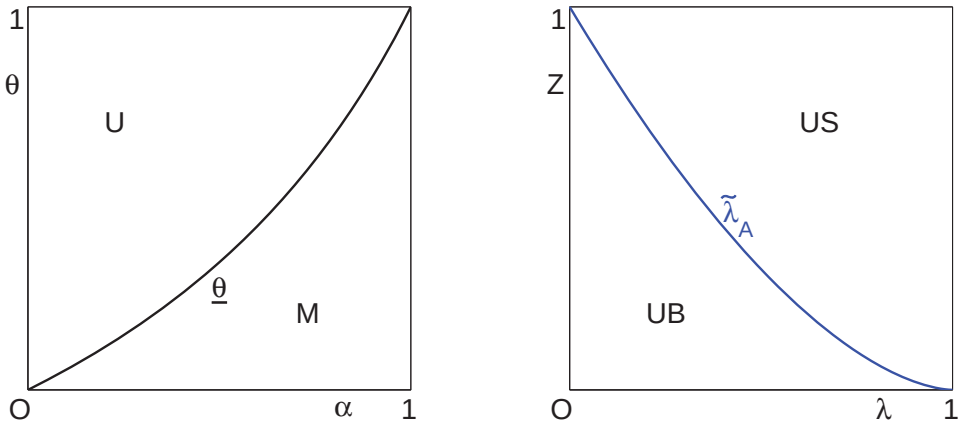

Figure 4: Threshold Values for the Autarkic Equilibrium

Figure 4 shows the two threshold values, i.e., $\underline{\theta}$ and $\tilde{\lambda}_A$, in the $\{\alpha, \theta\}$ space and in the $\{\lambda, Z\}$ space, respectively. For $\{\alpha, \theta\}$ in region **U** of the left panel and $\{\lambda, Z\}$ in region **UB** (**US**) of the right panel of figure 4, there exists a **u**nique steady state under autarky where $w_A < \bar{w}_A$ ($w_A > \bar{w}_A$) and the borrowing constraints are **b**inding (**s**lack).[15] In figure 5, the solid curves show respectively the laws of motion for wage in these two cases, while the dashed curves show those in the case of $\lambda = 1$. For $w_t \in (0, \bar{w}_A)$, the solid curve lies below the dashed curve and the gap reflects the efficiency losses, $(1 - \Gamma_t^\alpha) \left(\frac{w_t}{\rho}\right)^\alpha$.

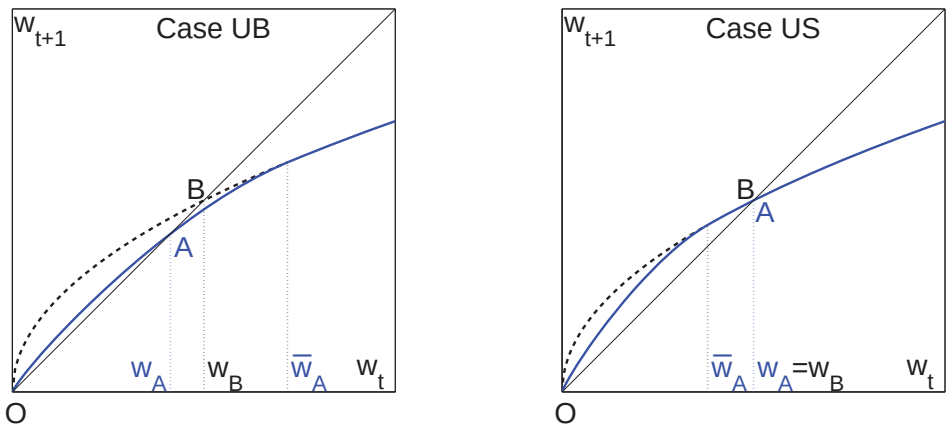

Figure 5: Laws of Motion for Wage under Autarky: $\theta \geq \underline{\theta}$

2.1 The World Economy under Autarkic Equilibrium

North and South are inherently identical, except that North is more financially developed and its population share in the world economy is negligible.

[15] See the proof of proposition 1 for the case of $\{\alpha, \theta\}$ in region M.

Assumption 1. $\theta \in (\underline{\theta}, 1)$, $0 < \lambda^* < \lambda < \tilde{\lambda}_A$, *and* $\frac{L}{L+L^*} \to 0$.[16]

According to proposition 1, assumption 1 ensures that, in each country, there is a unique, autarkic steady state where the borrowing constraints are binding.

Under autarky, the interest rate is coupled with the rate of return in sector 0,

$$r_t = q_{0,t+1} = \Upsilon_t[1 - \eta(1 - \mu_{t+1})] < \Upsilon_t. \tag{25}$$

Lemma 2. $w_A^* < w_A < w_B$, $\chi_A^* < \chi_A < 1$, $\mu_A^* < \mu_A < 1$, *and* $r_A^* < r_A < \rho$.

A higher level of financial development not only allows each entrepreneur to borrow and invest more, but also allows more agents to meet the MIR and invest in sector 1. The improvement in investment composition raises the aggregate investment efficiency. In period $t+1$, higher aggregate output implies higher aggregate saving, which further improves investment composition and production efficiency. In the long run, national income is higher and so is the sectoral price ratio.

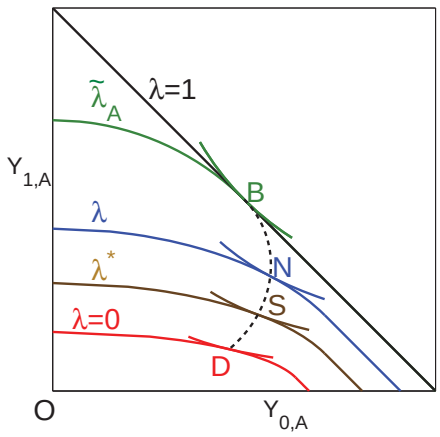

Figure 6: Steady-State Pattern of Sectoral Outputs under Autarky: $\lambda \in [0, 1]$

The dashed curve in figure 6 shows the steady-state sectoral outputs as the functions of $\lambda \in [0, \tilde{\lambda}_A)$. At the autarkic steady state, the (absolute) gradient of the PPF measures the sectoral output-price ratio, which rises in λ. This way, **the level of financial development is a determinant of comparative advantage** in this model, i.e., good 1 (0) is cheaper in North (South) at the autarkic steady state.

Financial development affects the interest rate in three ways. First, it allows each entrepreneur to borrow more as well as allows more agents to become entrepreneurs. The higher (lower) aggregate credit demand (supply) on the ***i***ntensive and ***e***xtensive ***m***argins leads to a higher interest rate. I call it the **IEM** effect. Second, by channeling domestic saving towards the high-return sector, financial development improves aggregate efficiency and raises the social rate of return, leading to a higher interest rate in period t. Third, higher aggregate output

[16] By definition, the composite parameter Z is independent of the level of financial development and the population size. Thus, Z takes the same value for both countries.

in period $t+1$ implies higher domestic saving and investment. Due to the decreasing MRK effect, the social rate of return declines in the long run and so does the interest rate.

At the autarkic steady state, the decreasing MRK effect and the aggregate efficiency effect cancel out.[17] Due to the positive **IEM** effect, the interest rate is higher in North than in South. Starting from the autarkic steady state, if agents are allowed to borrow and lend abroad[18], financial flows are "uphill" from South to North, which widens the cross-country income gap. In this model, the "global imbalances" arise as an equilibrium response to heterogeneous financial development across countries.

3 Can Trade Resolve the Global Imbalances?

The world economy is initially at the autarkic steady state. In period 0, the two countries announce that goods 0 and 1 will be freely traded from period 1 on.[19] Due to its negligible world population share, North is a small open economy and the equilibrium allocation in South does not respond to free trade. In the absence of trade costs, the sectoral price ratio in North is aligned to the world level from period $t=1$ on, $\chi_t = \chi^* = \chi_A^*$ and so is the sectoral return ratio, $\mu_t = \chi_t^{\frac{1}{\alpha}} = (\chi^*)^{\frac{1}{\alpha}} = \mu^*$.[20] Without international factor mobility, trade is balanced and the social rate of return is still characterized by equation (20).

3.1 Specialization as An Amplification Mechanism

Lemma 3. *If $w_t < \bar{w}$ ($w_t \geq \bar{w}$), North specializes partially (fully) in sector 1.*

Figure 7 shows the sectoral dynamics in North, where $P_t \equiv (Y_{0,t}, Y_{1,t})$ denotes domestic **p**roduction and $A_t \equiv (V_{0,t}, V_{1,t})$ denotes domestic **a**bsorption in the two sectors. In period 0, North is at the autarkic steady state with $A_0 = P_0$ and $MRS_{0,1} = MRT_{0,1} = \chi_A$. The announcement of free trade does not affect the sectoral investment in period 0, $\delta_0 = \left(\frac{w_0}{\bar{w}}\right)^{\frac{1-\theta}{\theta}} = \delta_A$, so that the PPF in period 1 is the same as before. By aligning the sectoral price ratio to the world level $\chi_1 = \chi^* < \chi_A$, trade induces North to export (import) good 1 (0) in period 1, while the rise (decline) in the MRL in sector 1 (0) drives the labor reallocation from sector 0 to 1, $\zeta_1 = \frac{1}{1+(\frac{1}{\delta_A}-1)(\chi^*)^{\frac{1}{\alpha}}} > \zeta_A$. Thus, domestic production moves from point P_0 to P_1.

[17] Due to inelastic aggregate saving $w_t L$ and the Cobb-Douglas production functions, the social rate of return at the autarkic steady state is constant at ρ, independent of λ. See equation (20). In von Hagen and Zhang (2014), aggregate saving is elastic and the extensive margin is mute. At the autarkic steady state, the decreasing MRK effect dominates the aggregate efficiency effect so that the social rate of return declines in λ. Nevertheless, the intensive-margin effect dominates so that the interest rate is still higher in the more financially developed country.

[18] The final good is tradeable and serves as the vehicle for international borrowing/lending.

[19] If free trade is announced and implemented in period 0, χ_0 is immediately aligned with χ_A^*, which unexpectedly affects the investment return of those born in period $t=-1$. In the two-period OLG setting, announcing the policy one period in advance avoids the uncertainty.

[20] Subsection 4.2 shows that the degree of specialization depends on trade costs.

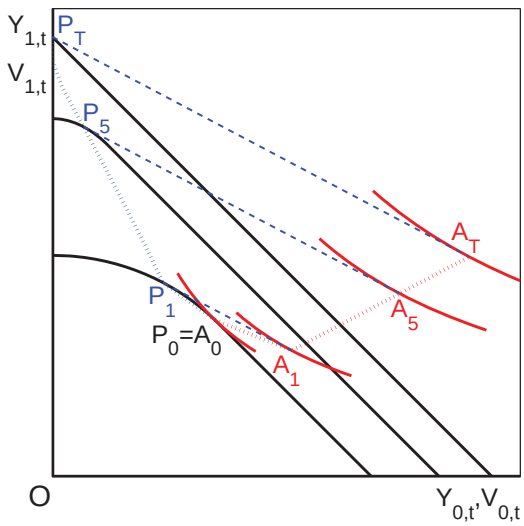

Figure 7: The Equilibrium Dynamics in North under Free Trade

Trade decouples domestic absorption from domestic production and the national income line (the dashed line linking P_t and A_t) is flatter than in period 0, $\chi_1 = \chi^* < \chi_0 = \chi_A$. The static gains from trade raise national income, as shown by the fact that domestic absorption A_1 is on the isoquant higher than A_0. It allows more agents to overcome the MIR and invest in sector 1, $\delta_1 = \left(\frac{w_1}{\bar{w}}\right)^{\frac{1-\theta}{\theta}} > \delta_0$, which enhances North's comparative advantage and induces it to specialize further towards sector 1 along the labor margin in period 2, $\zeta_2 = \frac{1}{1+(\frac{1}{\delta_1}-1)(\chi^*)^{\frac{1}{\alpha}}} > \zeta_1$.

This way, the static gains in period 1 trigger a dynamic, virtuous cycle, i.e., the rise in national income improves North's comparative advantage along the extensive margin of sectoral investment and the enhanced specialization in the next period raises national income further. I call it the *trade-driven investment reallocation effect*. As long as it dominates the decreasing MRK effect, such an amplification process propagates over time until the mass of entrepreneurs becomes so large that their borrowing capacity exceeds the entire household labor income and sector 0 vanishes in North, i.e., $w_t \geq \bar{w}$ and $\delta_t = 1$. From then on, the decreasing MRK effect brings North to a new steady state.

As explained in section 1, higher national income affects the PPF in the next period through the investment scale effect and the investment composition effect. The former moves the PPF parallel away from the origin, while the latter extends the linear part of the PPF by raising $\tilde{\delta}_t$, given $w_t \leq \bar{w}$. Let X_T denote the steady-state value of variable X_t under trade integration. In figure 7, the dotted line linking P_1 and P_T, i.e., the path of domestic sectoral production, eventually aligns with the vertical axis, implying that North offshores sector 0 at some point in time. The dotted line linking A_1 and A_T features the path of domestic sectoral absorption, showing the rise in national income over time.

The dynamic impacts of trade on national income can be illustrated by the law of motion

for wage, which is piecewise and characterized by equation (26),

$$w_{t+1} = \left(\frac{1}{\rho} w_t \Gamma_t\right)^{\alpha}, \ \Gamma_t = (\mu^*)^{\eta}[1 + (\frac{1}{\mu^*} - 1)\delta_t], \tag{26}$$

where $\delta_t = \left(\frac{w_t}{\bar{w}}\right)^{\frac{1-\theta}{\theta}} < 1$ for $w_t < \bar{w}$ and $\delta_t = 1$ for $w_t \geq \bar{w}$.

Proposition 2. *Under free trade, North may converge towards a **u**nique steady state where it offshores sector 0 and **f**ully specializes in sector **1**.*

Let case **UF1** denote the one described in proposition 2. It arises if the static gains in period 1 and the investment reallocation effect are sufficiently large.

- The larger the cross-country differences in financial development $\lambda - \lambda^*$, the larger the cross-country differences in the sectoral price ratio $\chi_A - \chi_A^*$, the larger the trade flows and the static gains in period 1.

$$\frac{\partial \ln w_1}{\partial \ln \chi_1} = \frac{\partial \ln \Gamma_0}{\partial \ln \mu_1} = \frac{\mu^* - \mu_A}{\frac{\mu^*}{\eta} + \frac{\mu_A}{1-\eta}}. \tag{27}$$

  Given $\mu^* = \mu_A^* < \mu_A$, $\frac{\partial \ln w_1}{\partial \ln \chi_1} < 0$. Thus, the static gains raise national income $w_1 > w_0$ by a *downward* alignment of $\mu_1 = \mu^* < \mu_A$.

- From period 1 on, national income is driven by the investment reallocation effect and the decreasing MRK effect if $w_t < \bar{w}$, while it is driven purely by the decreasing MRK effect if $w_t \geq \bar{w}$. The lower the λ^*, the lower the χ^* and the μ^*, the larger the sectoral price and rate-of-return differentials on the world market, the larger the gains for North by specializing in the high-return sector, the stronger the investment reallocation effect.

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - \underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} + \underbrace{\alpha \frac{\partial \ln \Gamma_t}{\partial \ln \delta_t} \frac{\partial \ln \delta_t}{\partial \ln w_t}}_{\text{investment reallocation effect}} = \alpha + \frac{\alpha \frac{\partial \ln \delta_t}{\partial \ln w_t}}{1 + \frac{1}{(\frac{1}{\mu^*} - 1)\delta_t}}. \tag{28}$$

- Given $\theta \geq \underline{\theta}$, the lower the θ, the less dispersed the wealth distribution, the more sensitive the mass of entrepreneurs and sectoral investment to the static gains and income changes, the stronger the investment reallocation effect.

$$\frac{\partial \ln \delta_t}{\partial \ln w_t} = \begin{cases} \frac{1-\theta}{\theta}, & \text{if } w_t < \bar{w}; \\ 0 & \text{if } w_t \geq \bar{w}. \end{cases} \tag{29}$$

Case UF1 arises for $\{\theta, Z, \lambda^*, \lambda\}$ in region UF1 of figure 8, i.e., $\theta \in [\underline{\theta}, \tilde{\theta})$, $\lambda^* < \tilde{\lambda}_T^*$, and $\lambda > \bar{\lambda}_T$ where the threshold value $\bar{\lambda}_T > \lambda^*$ increases in λ^*. In the following, I focus on case UF1 and analyze the dynamic responses of national income and the interest rate to trade.[21]

[21] Appendix A gives a complete analysis of the dynamic and steady-state properties under trade.

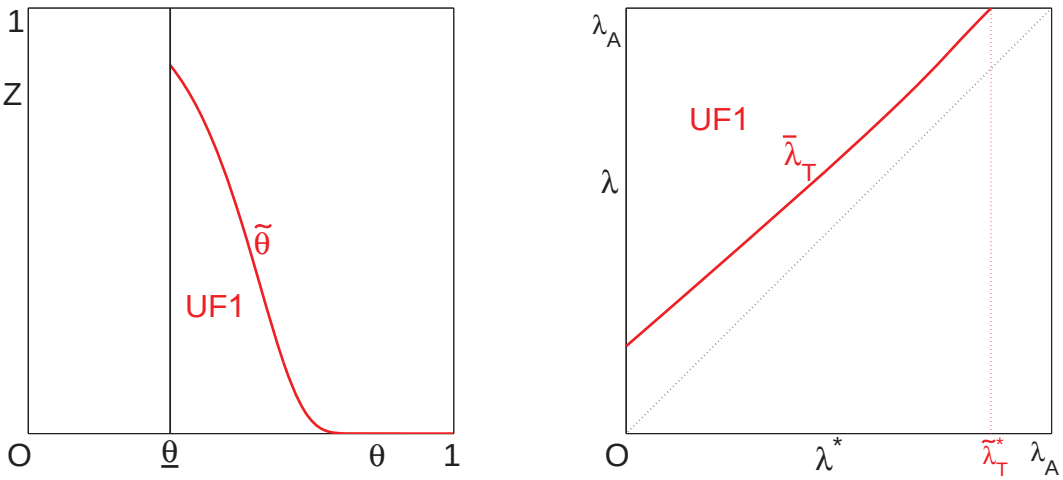

Figure 8: Threshold Values for the Free-Trade Equilibrium

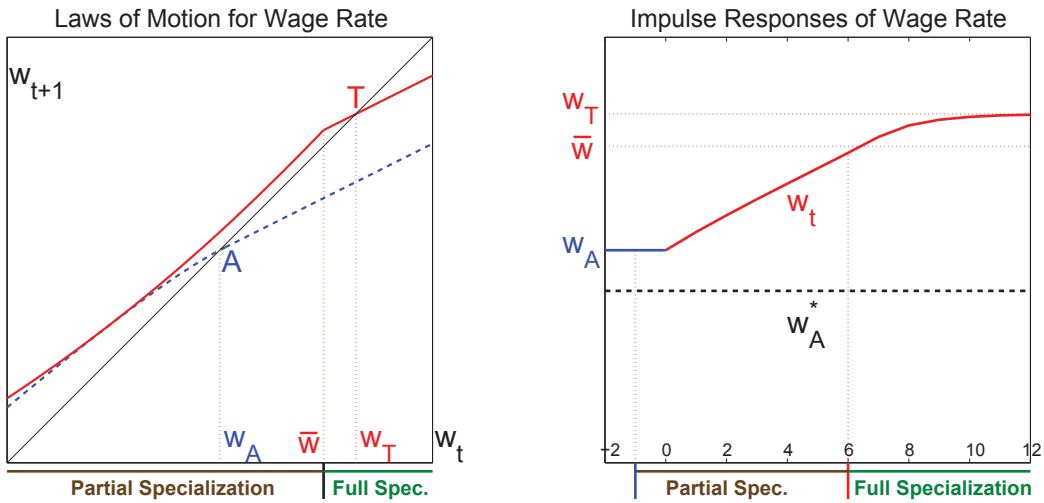

Figure 9: Dynamics of National Income: From Autarkic Steady State to Free Trade

In figure 9, the solid (dashed) curve in the left panel shows the law of motion for wage under trade (autarky), while the solid curve in the right panel shows the impulse responses of wage under trade.[22] For period $t \leq 6$, $w_t < \bar{w}$ and hence, $\delta_t < 1$. As the investment reallocation effect dominates the decreasing MRK effect, national income rises over time. For period $t \geq 7$, $w_t > \bar{w}$ and North offshores sector 0, $\delta_t = 1$. Then, the decreasing MRK effect brings North to steady state T.

3.2 Interest Rate Reversal and Re-reversal under Free Trade

Lemma 4. *Under trade, the interest rate in North is a piecewise function of national income, depending on whether the low-return sector is fully offshored.*

1.) For $w_t \geq \bar{w}$, the mass of entrepreneurs is so large that their total debt capacity exceeds the

[22]For illustration clarity, the axes in the left panel are scaled in logarithm and so is the vertical axis in the right panel. This scaling approach also applies to figures 10, 14, and 15.

entire household savings and the aggregate credit demand pushes the interest rate equal to the rate of return in sector 1,

$$r_t = q_{1,t+1} = \Upsilon_t = \frac{\rho w_{t+1}}{w_t}, \qquad \frac{\partial \ln r_t}{\partial \ln w_t} = -\underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} < 0. \tag{30}$$

The higher the national income, the higher the domestic saving and investment, the lower the MRK and the rate of return in sector 1, the lower the interest rate.

2.) For $w_t \in (0, \bar{w})$, the mass of entrepreneurs is so small that they cannot borrow the entire household labor income. Besides lending to entrepreneurs, households invest the rest of their labor income in sector 0. The no-arbitrage condition gives

$$\begin{aligned} r_t = q_{0,t+1} &= \frac{\Upsilon_t}{1 + \frac{(1-\mu^*)\delta_t}{\mu^*}}, \\ \frac{\partial \ln r_t}{\partial \ln w_t} &= -\underbrace{(1-\alpha)}_{\text{decreasing MRK effect}} - \underbrace{\frac{(1-\alpha)\left(\frac{1}{\theta}-1\right)}{1 + \frac{1}{(\frac{1}{\mu^*}-1)\delta_t}}}_{\text{reallocation effect}} < 0. \end{aligned} \tag{31}$$

In period 1, trade only triggers the labor reallocation, i.e., $\zeta_1 > \zeta_A$ and $\delta_0 = \delta_A$, leading to a higher capital-labor ratio in sector 0,

$$\frac{K_{0,1}}{L_{0,1}} = \frac{1-\delta_0}{1-\zeta_1}\frac{w_0 L}{L} = \left[1 + \delta_A(\frac{1}{\mu^*} - 1)\right] w_0 > \frac{K_{0,0}}{L_{0,0}} = \left[1 + \delta_A(\frac{1}{\mu_A} - 1)\right] w_A.$$

Meanwhile, the price of good 0 falls, $p_{0,1} = \chi_1^\eta = (\chi^*)^\eta < p_{0,0} = \chi_A^\eta$. Both effects reduce the rental price of capital in sector 0,

$$q_{0,1} = p_{0,1}\frac{\alpha Y_{0,1}}{K_{0,1}} = p_{0,1}\left(\frac{K_{0,1}}{\rho L_{0,1}}\right)^{\alpha-1} < q_{0,0} = p_{0,0}\left(\frac{K_{0,0}}{\rho L_{0,0}}\right)^{\alpha-1}.$$

Thus, the interest rate falls in period 0. I call it the reallocation effect. It dominates the **IEM** effect so that the interest rate in North is even lower than the world level, $r_0 < r_A^* < r_A$, a result Antras and Caballero (2009) calls **the interest rate reversal**.

From period $t = 1$ on, trade triggers sectoral reallocation of labor and investment. Labor reallocation is frictionless, while it is not the case for investment. Besides the scale effect, the rise in national income also raises the capital-labor share in sector 0 through the disproportional sectoral reallocation of labor and investment. Use equation (6) to get

$$\frac{K_{0,t+1}}{L_{0,t+1}} = \frac{1-\delta_t}{1-\zeta_{t+1}}\frac{w_t L}{L} = \underbrace{\left[1 + \delta_t\left(\frac{1}{\mu^*} - 1\right)\right]}_{\text{reallocation effect}} \underbrace{w_t}_{\text{scale effect}}.$$

The reallocation effect and the decreasing MRK effect jointly reduce the MRK in sector 0, leading to the lower interest rate over time.

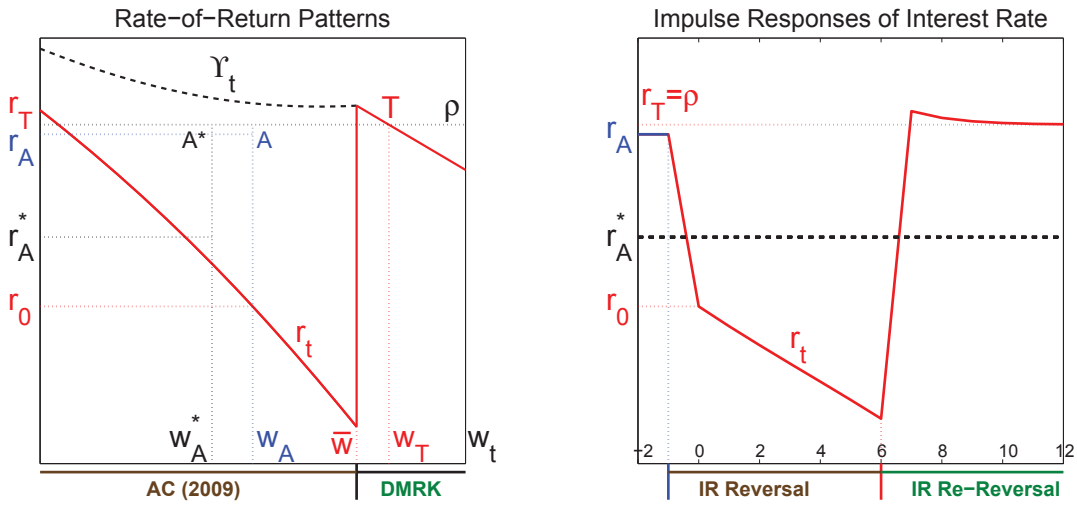

Figure 10: The Interest Rate Dynamics: From Autarkic Steady State to Free Trade

The left panel of figure 10 shows the interest rate as a piecewise function of national income, while the right panel shows the interest rate responses to trade. In period 0, the announcement of free trade leads to the interest rate reversal, $r_0 < r_A^* < r_A$. From period 1 on, the factor reallocation effect and the decreasing MRK effect reduce the interest rate further. This process continues as long as sector 0 is active. In period 7, North fully offshores sector 0 and the interest rate is decoupled from (coupled with) the rate of return to sector 0 (1), shown by an interest rate jump in the right panel. I call it **the interest rate re-reversal**. The decreasing MRK effect then reduces the interest rate over time until North reaches its new steady state with $r_T = \Upsilon_T = \rho > r_A > r_A^*$.[23]

Proposition 3. *In case UF1, North may witness a non-monotonic interest rate pattern along the convergence path under trade. In the new steady state, the interest rate is higher than under autarky,* $r_T > r_A > r_A^*$.

For the parameter configurations outside region UF1 of figure 8, the static gains and the investment reallocation effect are too weak to ensure that North offshores sector 0. In that case, trade leads to the interest rate reversal, as predicted by Antras and Caballero (2009). To sum up, whether trade lead to the interest rate re-reversal depends on how far it reshapes North's sectoral composition.

3.3 FPE and The Timing of Economic Integration

In Antras and Caballero (2009), financial frictions distort domestic allocation in two dimensions. In the intratemporal dimension, the deviation of the sectoral price ratio from its efficient

[23] When North specializes fully in sector 1, the borrowing constraints are slack in North. Section 4.3 generalizes this two-sector setting into a multi-sector setting. When trade induces North to offshore the lowest-return sector, the interest rate is coupled with the rate of return in the second lowest-return sector, while the borrowing constraints are still binding in all other sectors.

level $(1-\chi_t)$ reflects the distortion on the cross-sector investment allocation; in the intertemporal dimension, the deviation of the interest rate from the social rate of return $(1-\frac{r_t}{\Upsilon_t})$ reflects the distortion on the aggregate credit demand. Trade (capital) flows alone only equalize the intratemporal (intertemporal) distortion across countries. Either case, factor prices (i.e., the wage rate, the interest rate, etc) do not equalize across countries. If trade and capital flows are allowed simultaneously, the distortions are equalized across countries in both dimensions, leading to the cross-country equalization of the factor prices as well as income per capita.

The logic is as follows. According to the right panels of figure 9 and 10, trade induces North to specialize *partially* towards sector 1 until period 6, while the interest rate reversal happens. If agents can borrow/lend abroad before or in period 6, financial flows are "downhill" from North to South and the interest rate in North is aligned to the world level, $r_t = r_t^*$. Due to the interest rate coupling, the rate of return in sector 0 is also equalized, $q_{0,t+1} = r_t = r_t^* = q_{0,t+1}^*$. Besides, by equalizing the sectoral price ratio $\chi_t = \chi_t^*$, trade implicitly equalizes the sectoral rental-price-of-capital ratio, $\mu_t = \chi_t^{\frac{1}{\alpha}} = (\chi_t^*)^{\frac{1}{\alpha}} = \mu_t^*$. This way, trade and capital mobility jointly equalize the rate of return in sector 1, $q_{1,t} = \frac{q_{0,t}}{\mu_t} = \frac{q_{0,t}^*}{\mu_t^*} = q_{1,t}^*$. Despite international labor immobility, the wage rate is equalized across countries and so is income per capita,[24]

$$\frac{Y_t}{2L} = \frac{w_t}{2(1-\alpha)} = \frac{(q_{0,t}^{1-\eta} q_{1,t}^{\eta})^{-\rho}}{2(1-\alpha)} = \frac{[(q_{0,t}^*)^{1-\eta}(q_{1,t}^*)^{\eta}]^{-\rho}}{2(1-\alpha)} = \frac{w_t^*}{2(1-\alpha)} = \frac{Y_t^*}{2L^*}.$$

Here, the factor price equalization (FPE, hereafter) and cross-country income convergence depend on the fact that trade only leads to partial specialization and hence, the interest rate is coupled with the sector 0's rate of return before and in period 6.

Suppose that agents are allowed to borrow/lend abroad only from period 7 on. As North has offshored sector 0, the interest rate is decoupled from the rate of return there. Thus, the interest rate equalization does not imply the cross-country equalization of sector 0's rate of return, which invalidates the aforementioned mechanism. In fact, the interest rate is re-reversed from period $t \geq 7$ on. If allowed, financial flows are from South to North, which widens the cross-country income gap. It is in stark contrast to the findings of Antras and Caballero (2009).

To sum up, whether trade and capital mobility jointly lead to the FPE depends critically on the **timing of integration**, i.e., whether the more financially developed country has fully offshored the low-return sector upon financial integration.

4 Discussions and Extensions

The endogenous response of δ_t to the static gains is a driving force that induces North to offshore sector 0, leading to the interest rate re-reversal. In this section, I first compare two channels through which the static gains from trade affect δ_t and then discuss the roles of trade costs

[24] In period t, each country is populated with the young and the old of the same mass. Thus, the size of total population is $2L$ in North and $2L^*$ in South, respectively.

and the initial conditions in determining the international sectoral price differentials. Finally, I show in a three-sector model that the interest rate reversal and re-reversal may arise recurrently if trade induces North to sequentially offshore the low-return sectors.

4.1 Endogenous Responses of δ_t through Two Channels

Let W_t^e and W_t denote respectively entrepreneurial wealth and national wealth. In the case of the binding borrowing constraints, the fraction of domestic investment in sector 1, $\delta_t = \frac{\frac{W_t^e}{1-\lambda}}{W_t} = \frac{1}{1-\lambda}\frac{W_t^e}{W_t}$, is proportional to the leverage multiplier $\frac{1}{1-\lambda}$ and the entrepreneurial wealth share $\frac{W_t^e}{W_t}$. The former is constant by assumption, while the latter is higher if the mass of entrepreneurs is higher (the **extensive** margin) and/or if the average wealth of entrepreneurs rises relative to the national average (the **intensive** margin). As shown below, δ_t may respond to the static gains from trade through these two margins.

In my model, agents live for **two** periods; each agent invests its entire labor income when young and consumes when old. Due to the absence of wealth accumulation at the individual level, agent j's wealth relevant for investment is just its labor income, $n_{j,t} = w_t(1-\theta)\epsilon_j$. One can decompose along two margins the impacts of the static gains from trade on the entrepreneurial wealth share in North,

$$\begin{aligned}\frac{W_1^e}{W_1} - \frac{W_A^e}{W_A} &= \underbrace{\frac{\int_{\underline{\epsilon}_A}^{\infty} n_{j,1} dG(\epsilon_j)}{w_1 L} - \frac{\int_{\underline{\epsilon}_A}^{\infty} n_{j,A} dG(\epsilon_j)}{w_A L}}_{\text{the intensive margin effect}} + \underbrace{\frac{\int_{\underline{\epsilon}_1}^{\underline{\epsilon}_A} n_{j,1} dG(\epsilon_j)}{w_1 L}}_{\text{the extensive margin effect}} \\ &= \underbrace{\frac{w_1 L \tau_A^{1-\theta}}{w_1 L} - \frac{w_A L \tau_A^{1-\theta}}{w_A L}}_{\text{the intensive margin effect (0)}} + \underbrace{\frac{w_1 L(\tau_1^{1-\theta} - \tau_A^{1-\theta})}{w_1 L}}_{\text{the extensive margin effect (+)}} .\end{aligned} \tag{32}$$

Consider the agents who are born in period 1 and would meet the MIR at the autarkic steady state, i.e., $\epsilon_j \geq \underline{\epsilon}_A$. The static gains affect in equal proportions the wealth of these "existing" entrepreneurs,[25] $n_{j,t} = w_t(1-\theta)\epsilon_j$, and the national wealth, $w_t L$, through the wage rate. Since the static gains from trade do not affect the national wealth share of "existing" entrepreneurs, the intensive margin is mute. Due to financial frictions and sector-specific MIR, the cutoff value $\underline{\epsilon}_t$ and the mass of entrepreneurs $\tau_t = \underline{\epsilon}_t^{-\frac{1}{\theta}}$ are endogenous. Thus, the static gains raise $\frac{W_t^e}{W_t}$ purely through the extensive margin, which then raises $\delta_t = \frac{1}{1-\lambda}\frac{W_t^e}{W_t}$ and creates the possibility of North offshoring sector 0.

In the static model of Antras and Caballero (2009), the extensive margin of entrepreneurial wealth is mute, due to two assumptions: **(1)** there is no MIR; **(2)** only a fixed mass τ of agents can invest in the constrained sector and are called entrepreneurs. In the case of the homogeneous labor endowment, $l_j = 1$, one can embed this static model into the two-period OLG

[25] Referring these agents as "existing" entrepreneurs is purely for ease of exposition. It may be misleading, because they are born in period 1 and did not "exist" in period 0.

setting and the wealth share of entrepreneurs is constant at $\frac{W_t^e}{W_t} = \frac{\tau w_t L}{w_t L} = \tau$. Under assumption 1 of Antras and Caballero (2009), the borrowing constraints are so tight that the investment share of the constrained sector is inefficiently low, $\delta = \frac{\tau}{1-\lambda} < \eta$. As δ is constant and does not respond to the static gains from trade, the unconstrained sector is always active, $\frac{K_{0,t+1}}{w_t L} = 1 - \delta > 1 - \eta > 0$ and the interest rate re-reversal never happens. It confirms that **the endogenous extensive margin of entrepreneurial wealth** in my two-period OLG model is key to the interest rate re-reversal.

Antras and Caballero (2009) embed their static model into a continuous-time setting with two key assumptions: **(1)** agents are born at a constant rate per unit of time and die at the same rate, **(2)** agents save all their (labor and investment) income and consume only when they die. Due to the law of large numbers, the mass of entrepreneurs is constant and hence, the extensive margin of entrepreneurial wealth is mute. Different from the two-period OLG setting, agents accumulate wealth over their lifetime. Due to the privilege of investing in the constrained sector at a higher rate of return, entrepreneurs accumulate wealth at a faster speed than others under autarky and hence, their wealth share $\frac{W_t^e}{W_t}$ becomes endogenous on the intensive margin. By aligning the sectoral price ratio at the world level, trade raises (reduces) the sectoral rate-of-return differential in North (South), which induces entrepreneurs to accumulate wealth at a faster (slower) speed than in the autarkic steady state. In this case, trade affects the entrepreneurial wealth share along **the intensive margin** and δ_t responds accordingly.

Antras and Caballero (2009) focus on the interest rate response to trade flows for the **less** financially developed country. As both sectors are always active there, the interest rate reversal always holds. They do not analyze explicitly whether and under what conditions trade can induce the **more** financially developed country to offshore the low-return sector.[26] In contrast, I highlight explicitly the **extensive margin** as a key channel through which the entrepreneurial wealth share responds to the static gains and show that trade may lead to the interest rate re-reversal if it substantially changes the sectoral composition in the **more** financially developed country.[27] This result complements the findings of Antras and Caballero (2009).

4.2 The Implications of Trade Costs and Initial Conditions

So far, I have made two simplifying assumptions: **(1)** there is no trade cost, and **(2)** the world economy is initially at the autarkic steady state. Given $\{\theta, Z\}$ in region UF1 of the left panel of figure 8, the left panel of figure 11 shows that case UF1 arises if $\chi^* < \tilde{\chi}_T^*$ and $\chi_A > \bar{\chi}_T$. Let $\underline{\chi}_A$ denote the sectoral price ratio at the autarkic steady state in the case of $\lambda = 0$.[28] It holds that $\chi_A, \chi_A^* \geq \underline{\chi}_A$.

[26] It is beyond the scope of my paper to explore this possibility in their dynamic model.

[27] Allowing wealth accumulation at the individual level in my model would strengthen my results through the additional, positive intensive margin effect.

[28] It is the (absolute) gradient of the PPF at point Z in figure 6.

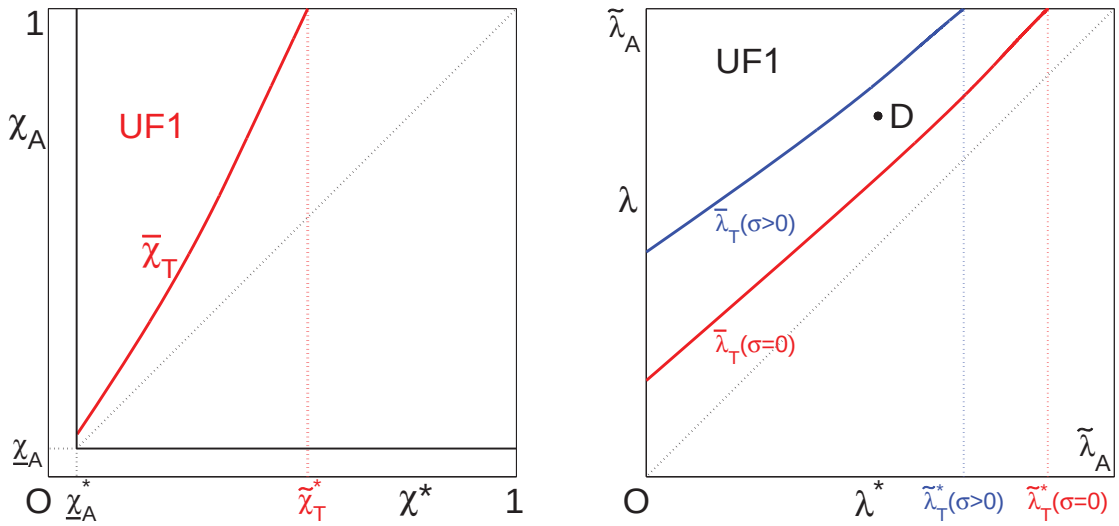

Figure 11: Threshold Values for the Trade Equilibrium with Trade Costs $\sigma \in (0, \bar{\sigma})$

Under the two assumptions, the world sectoral price ratio that North faces is $\chi^* = \chi_A^*$. Then, one can convert the threshold values ($\tilde{\chi}_T^*$ and $\bar{\chi}_T$) into the $\{\lambda^*, \lambda\}$ space, as shown by the right panel of figure 8. In the following, I discuss how trade costs and South's initial condition affect the likelihood of case UF1 for North.

Consider the trade costs first. Suppose that the world economy is initially at the autarkic steady state. Let $\sigma \in (0, 1)$ denote the proportional trade costs. Although trade costs do not affect the autarkic relative price in each country, they raise the world sectoral price ratio that North faces under trade $\chi^* = \chi_A^* \frac{1+\sigma}{1-\sigma}$, which narrows the international sectoral price differential, $\chi_A - \chi^* = \chi_A - \chi_A^* \frac{1+\sigma}{1-\sigma} < \chi_A - \chi_A^*$, and reduces the trade flows in period 1. This way, trade costs weaken the static gains from trade and the subsequent investment reallocation effect. For the existence of case UF1, χ_A^* needs to be lower than in the absence of trade costs ($\sigma = 0$) and so does λ^*. In the right panel of figure 11, trade costs reduce the threshold value $\tilde{\lambda}_T^*$ and raise the threshold value $\bar{\lambda}_T$. The larger the trade costs, the smaller the region UF1 in the right panel of figure 11, the less likely case UF1 can arise.

Lemma 5. *Let* $\bar{\sigma} \equiv \frac{\eta^{\frac{(1-\alpha)\theta}{(1-\eta)(1-\theta)}} - \underline{\chi}_A}{\eta^{\frac{(1-\alpha)\theta}{(1-\eta)(1-\theta)}} + \underline{\chi}_A}$. *For* $\sigma \in (0, \bar{\sigma})$, *a sufficiently large decline in trade costs may lead to a non-monotonic interest rate dynamics in North.*

Take as an example the parameter configuration represented by point D in the right panel of figure 11. For the sufficiently large trade costs, point D is outside of region UF1 and trade does not induce North to offshore sector 0. A marginal decline in trade costs reduces $\chi^* = \chi_A^* \frac{1+\sigma}{1-\sigma}$ and widens the international sectoral price differential, $\chi_A - \chi^* = \chi_A - \chi_A^* \frac{1+\sigma}{1-\sigma}$, which raises the trade flows and allows North to specialize further towards sector 1. Meanwhile, the decline in trade costs reduces the threshold value $\bar{\lambda}_T$ and shifts rightwards the border of region UF1, making case UF1 more likely. If the border is far above point D, a marginal decline in trade costs reduces χ^* and the interest rate in North falls, as mentioned in subsection 3.2. However,

if the decline in trade costs is so large that the border of region UF1 is below point D, North eventually offshores sector 0, leading to the interest rate re-reversal. This way, deepening trade integration may have the non-monotonic effect on the interest rate in North.

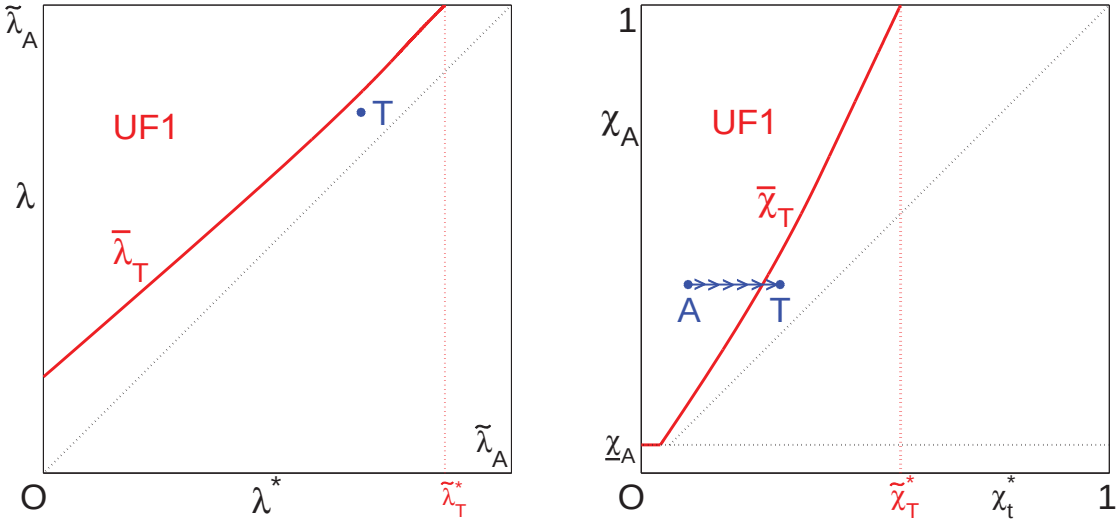

Figure 12: Threshold Values for the Trade Equilibrium with $w_0^* < w_A^*$

Next, consider the role of South's initial conditions. Suppose that there is no trade cost $(\sigma = 0)$ and North is initially at the autarkic steady state $(w_0 = w_A)$ as before, while South is below its autarkic steady state $(w_0^* < w_A^*)$. It is a relevant case for many emerging economies, e.g., China and Emerging Asia. Take as an example the parameter configuration represented by point T in the left panel of figure 12. Since point T is outside of region UF1, trade integration with South does not induce North to offshore sector 0, if South is initially at the autarkic steady state. However, if South is initially below the autarkic steady state, $w_0^* < w_A^*$ implies $\mu_1^* < \mu_A^*$ and $\chi_1^* < \chi_A^*$, according to equations (12) and (23). Then, case UF1 may arise for North in the short run.

The right panel of figure 12 shows the threshold values in the $\{\chi_t^*, \chi_A\}$ space, while point A and T denote the sectoral price ratio in South in period 1 and in the long run, respectively. Starting with point A, trade integration with South may induce North to gradually offshore sector 0 in the medium run if South converges slowly towards its steady state[29] and χ_t^* stays in region UF1 for a sufficiently long time. However, when South is close to its steady state, χ_t^* is so high that trade flows become small and sector 0 becomes active again in North. In this sense, the economic development in South may affect the sectoral composition in North through trade linkages, which may have significant implications to the global imbalances.

[29]Under assumption 1, $L^* >> L$ so that trade integration with North does not affect the income dynamics of South. In the long run, South still converges to its autarkic steady state.

4.3 Recurrent Interest Rate Reversal and Re-Reversal

So far, I have shown that the interest rate re-reversal may occur when trade allows North to fully offshore the low-return sector. The two-sector setting helps explore the core mechanism. In this section, I embed it into a three-sector setting and show that the interest rate reversal and re-reversal may arise *recurrently* when trade allows North to *sequentially* offshore the low-return sectors.

Suppose that each country has three sectors, indexed by $s \in \{0, 1, 2\}$ and ranked according to the MIR, i.e., $\mathfrak{m}_2 > \mathfrak{m}_1 > \mathfrak{m}_0 = 0$. Let $\delta_{s,t}$ and $\zeta_{s,t+1}$ denote the respective shares of domestic saving $w_t L$ and labor input L allocated for the production of good s in period $t+1$, $Y_{s,t+1} = \left(\frac{\delta_{s,t} w_t L}{\alpha}\right)^{\alpha} \left(\frac{\zeta_{s,t+1} L}{1-\alpha}\right)^{1-\alpha}$. Sectoral outputs are combined to produce final goods, $Y_{t+1} = \Pi_{s=0}^{2} \left(\frac{V_{s,t+1}}{\eta_s}\right)^{\eta_s}$, where η_s denotes the sectoral share in the final good production and $\sum_{s=0}^{2} \eta_s = 1$. Let $\mu_{s,t} \equiv \frac{q_{s,t}}{q_{2,t}}$ and $\chi_{s,t} \equiv \frac{p_{s,t}}{p_{2,t}}$ denote respectively the relative rental price of capital and the relative output price of sector s with respect to those of sector 2.

Let $\underline{\epsilon}_{s,t} \equiv \frac{1-\lambda}{w_t} \frac{\mathfrak{m}_s}{1-\theta}$ and $\bar{w}_s \equiv \frac{\mathfrak{m}_s}{1-\theta}(1-\lambda)^{\frac{1}{1-\theta}}$. According to equation (11), the agents with $\epsilon_j \geq \underline{\epsilon}_{s,t}$ can meet the MIR and invest in sector $v \geq s$. According to equation (12), for $w_t \geq \bar{w}_s$, if the agents with $\epsilon_j \geq \underline{\epsilon}_{s,t}$ borrow and invest to the limit, domestic saving is fully invested in sector $v \geq s$, and sector $v < s$ is inactive.

4.3.1 Autarkic Equilibrium

As labor is perfectly mobile across sectors, the sectoral labor input is efficient at $\zeta_{s,t+1} = \eta_s$ under autarky. In the absence of the borrowing constraints, the same would also apply to the sectoral investment input, $\delta_{s,t} = \zeta_{s,t+1} = \eta_s$ and the cross-sector equalization of the capital-labor ratio $\frac{\delta_{s,t} w_t L}{\zeta_{s,t+1} L} = w_t$ would lead to the sectoral rate-of-return equalization, $q_{s,t+1} = r_t$. As shown below, the borrowing constraints may depress the mass of agents investing in the MIR sectors and the sectoral investment distortions then lead to the sectoral rate-of-return differentials.

Assumption 2. $\gamma \equiv \frac{\mathfrak{m}_1}{\mathfrak{m}_2} < \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{\frac{\theta}{1-\theta}}$.

Let $\bar{w}_{1,A} \equiv \frac{\bar{w}_1}{\left(1+\frac{\eta_0}{\eta_1}\left[1-\gamma^{\frac{1-\theta}{\theta}}\right]\right)^{\frac{\theta}{1-\theta}}} < \bar{w}_1$ and $\bar{w}_{2,A} \equiv \bar{w}_2 \eta_2^{\frac{\theta}{1-\theta}} < \bar{w}_2$.

For a sufficiently low level of national income $w_t < \bar{w}_{1,A}$, the mass of agents investing in sector 1 and 2 is inefficiently low and so is the aggregate credit demand. Thus, the rate-of-return spread, $r_t < q_{s,t+1}$, arises in sector $s \in \{1, 2\}$ and the borrowing constraints are binding

there. The sectoral investment shares are

$$\delta_{2,t} = \frac{\int_{\underline{\epsilon}_{2,t}}^{\infty} \frac{(1-\theta)\epsilon_j w_t}{1-\lambda} dG(\epsilon_j)}{w_t L} = \left(\frac{w_t}{\bar{w}_2}\right)^{\frac{1-\theta}{\theta}}, \tag{33}$$

$$\delta_{1,t} = \frac{\int_{\underline{\epsilon}_{1,t}}^{\underline{\epsilon}_{2,t}} \frac{(1-\theta)\epsilon_j w_t}{1-\lambda} dG(\epsilon_j)}{w_t L} = \left(\frac{w_t}{\bar{w}_1}\right)^{\frac{1-\theta}{\theta}} - \left(\frac{w_t}{\bar{w}_2}\right)^{\frac{1-\theta}{\theta}}, \tag{34}$$

$$\delta_{0,t} = 1 - (\delta_{1,t} + \delta_{2,t}) = 1 - \left(\frac{w_t}{\bar{w}_1}\right)^{\frac{1-\theta}{\theta}}. \tag{35}$$

Assumption 2 and $w_t < \bar{w}_{1,A}$ jointly ensure the descending sectoral capital-labor ratio, which implies the ascending pattern of the sectoral rate of return.

$$\frac{\delta_{0,t}}{\eta_0} > \frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}, \quad \Rightarrow \quad r_t = q_{0,t+1} < q_{1,t+1} < q_{2,t+1}. \tag{36}$$

As all sectoral outputs are essential for the production of final goods, all sectors are active under autarky and the interest rate is determined by the lowest-return sector (sector 0). A rise in national income raises the investment shares of sector 1 and 2 in equal proportions through the extensive margin, while that of sector 0 falls.[30]

Proposition 4. *Let* $\underline{\theta}_3 \equiv \frac{\alpha}{\alpha + \frac{1-\alpha}{\eta_1+\eta_2}}$, $\mathbb{K} \equiv \left(\frac{\gamma^{\frac{1-\theta}{\theta}}}{\eta_2\left[1+\frac{\eta_0}{\eta_1}\left(1-\gamma^{\frac{1-\theta}{\theta}}\right)\right]}\right)^{1-\frac{\theta}{\rho(1-\theta)}} \left(\frac{\eta_2(\gamma^{-\frac{1-\theta}{\theta}}-1)}{\eta_1}\right)^{1-\eta_2}$, *and* $Z_3 \equiv \frac{\frac{1}{\rho}}{\left(\frac{\mathrm{m}_2}{1-\theta}\eta_2^{\frac{\theta}{1-\theta}}\right)^{\frac{1}{\rho}}}$. *For* $\theta \in (\underline{\theta}_3, 1)$, $Z_3 < \frac{1}{\mathbb{K}}$, *and* $\lambda < \tilde{\lambda}_{1,A} \equiv 1 - (\mathbb{K}Z_3)^{\rho(1-\theta)}$, *there is a unique, autarkic steady state where the borrowing constraints are binding in sector 1 and 2.*

Figure 13 shows graphically proposition 4. Given $\{\alpha, \theta\}$ in region U of the left panel, for $\{\lambda, Z_3\}$ in region UB12 of the right panel, there is a **u**nique, autarkic steady state where the borrowing constraints are **b**inding in sector **1** and **2**. Let $X_{s,A}$ denote the autarkic steady-state value of variable $X_{s,t}$.

Lemma 6. *Given* $\theta > \underline{\theta}_3$ *and* $Z_3 < \frac{1}{\mathbb{K}}$, *for* $0 < \lambda^* < \lambda < \tilde{\lambda}_{1,A}$, $\chi^*_{0,A} < \chi_{0,A} < \chi^*_{1,A} = \chi_{1,A} < 1$, $\mu^*_{0,A} < \mu_{0,A} < \mu^*_{1,A} = \mu_{1,A} < 1$, *and* $r^*_A < r_A < \rho$.

[30] For a moderate level of national income $w_t \in (\bar{w}_{1,A}, \bar{w}_{2,A})$, the mass of agents investing in sector 2 is still so low that the rate-of-return spread, $r_t < q_{2,t+1}$, leads to the binding borrowing constraints and the investment share of sector 2 is still specified by equation (33). In contrast, the mass of agents investing in sector 1 is so large that the rate-of-return spread vanishes $r_t = q_{1,t+1}$ and the borrowing constraints are slack. Thus, the rate of return equalizes in sector 0 and 1, $q_{1,t+1} = r_t = q_{0,t+1}$, and so does the capital-labor ratio, $\frac{\delta_{1,t}}{\eta_1} = \frac{\delta_{0,t}}{\eta_0} = \frac{1-\delta_{2,t}}{\eta_0+\eta_1}$. A rise in national income raises the investment share of sector 2 through the extensive margin, while that of sector 0 and 1 falls in equal proportions.

For a high level of national income $w_t > \bar{w}_{2,A}$, the mass of agents investing in sector 1 and 2 are large enough to ensure the zero rate-of-return spread, i.e., $q_{s,t+1} = r_t$ for $s \in \{1, 2\}$. Thus, the sectoral investment shares are efficient at $\delta_{s,t} = \eta_s$, independent of the change in national income.

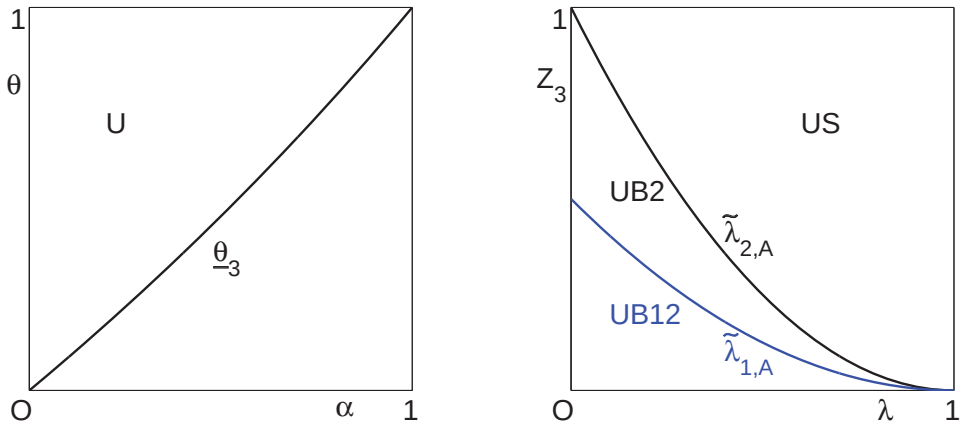

Figure 13: Threshold Values for the Autarkic Steady State in the 3-Sector Setting

At the autarkic steady state, the borrowing constraints are binding in sector $s \in \{1, 2\}$. According to equations (33)-(34), the relative capital-labor ratio of sector 1 is independent of the level of financial development, $\frac{\frac{\delta_{1,A}}{\eta_1}}{\frac{\delta_{2,A}}{\eta_2}} = \left(\gamma^{-\frac{1-\theta}{\theta}} - 1\right)\frac{\eta_2}{\eta_1} > 1$. Thus, $\mu_{1,A} = \mu^*_{1,A} < 1$ and $\chi_{1,A} = \chi^*_{1,A} < 1$. Besides, the higher the level of financial development, the less severe the overinvestment in sector 0, the higher the relative output price and the relative rate of return in sector 0, i.e., $\chi^*_{0,A} < \chi_{0,A}$ and $\mu^*_{0,A} < \mu_{0,A}$. Thus, South has a comparative advantage in sector 0, while North has a comparative advantage in sector $s \in \{1, 2\}$.

4.3.2 Free-Trade Equilibrium

The world economy is at the autarkic steady state when the free trade policy is announced in period 0. According to lemma 6, $\mu^*_{0,A} < \mu^*_{1,A} < \mu^*_{2,A} = 1$. From period $t = 1$ on, the relative sectoral prices in North are aligned to the world levels, $\chi_{s,t} = \chi^*_{s,A}$, and so are its relative sectoral rates of return, $\mu_{s,t} = \mu^*_{s,A}$, implying that the sectoral rate of return is strictly ascending in North, $q_{0,t+1} = \mu^*_{0,A} q_{2,t+1} < q_{1,t+1} = \mu^*_{1,A} q_{2,t+1} < q_{2,t+1}$. Thus, agents in North always invest in the sector with the highest MIR that they can afford.

The static gains from trade raises the national income in North, which then triggers the sectoral investment reallocation along the extensive margin.

For $w_t < \bar{w}_1$, the mass of agents investing in sector 1 and 2 is so low that they cannot borrow and invest the entire domestic saving there. Thus, sector 0 is active and the interest rate is coupled with the rate of return there, $r_t = q_{0,t+1}$. Given the ascending sectoral rates of return, the borrowing constraints are binding in sector 1 and 2. Thus, the sectoral investment shares are still featured by equations (33)-(35).

For $w_t \in (\bar{w}_1, \bar{w}_2)$, the mass of agents investing in sector 1 and 2 is so high that the entire domestic saving is invested in sector 1 and 2, while sector 0 is inactive, $\delta_{0,t} = 0$. The interest rate is then coupled with the rate of return in sector 1, $r_t = q_{1,t+1}$, and the borrowing constraints are slack there. Combine it with the ascending sectoral rate of return $q_{1,t+1} < q_{2,t+1}$ to get the binding borrowing constraints in sector 2. Thus, $\delta_{2,t}$ is still featured by equation (33), while

$\delta_{1,t} = 1 - \delta_{2,t}$.

For $w_t > \bar{w}_2$, the mass of agents investing in sector 2 is so large as to fully absorb domestic saving $\delta_{2,t} = 1$, while other sectors are inactive $\delta_{0,t} = \delta_{1,t} = 0$.

As shown in subsection 3.2, whether trade leads to the interest rate re-reversal depends critically on the magnitude of the static and the dynamic gains from trade and, ultimately, on the cross-country heterogeneity in financial development.

Proposition 5. *For a sufficiently large heterogeneity in financial development, trade integration induces North to offshore sequentially the low-MIR, low-return sectors and upgrade gradually to the high-MIR, high-return sectors.*

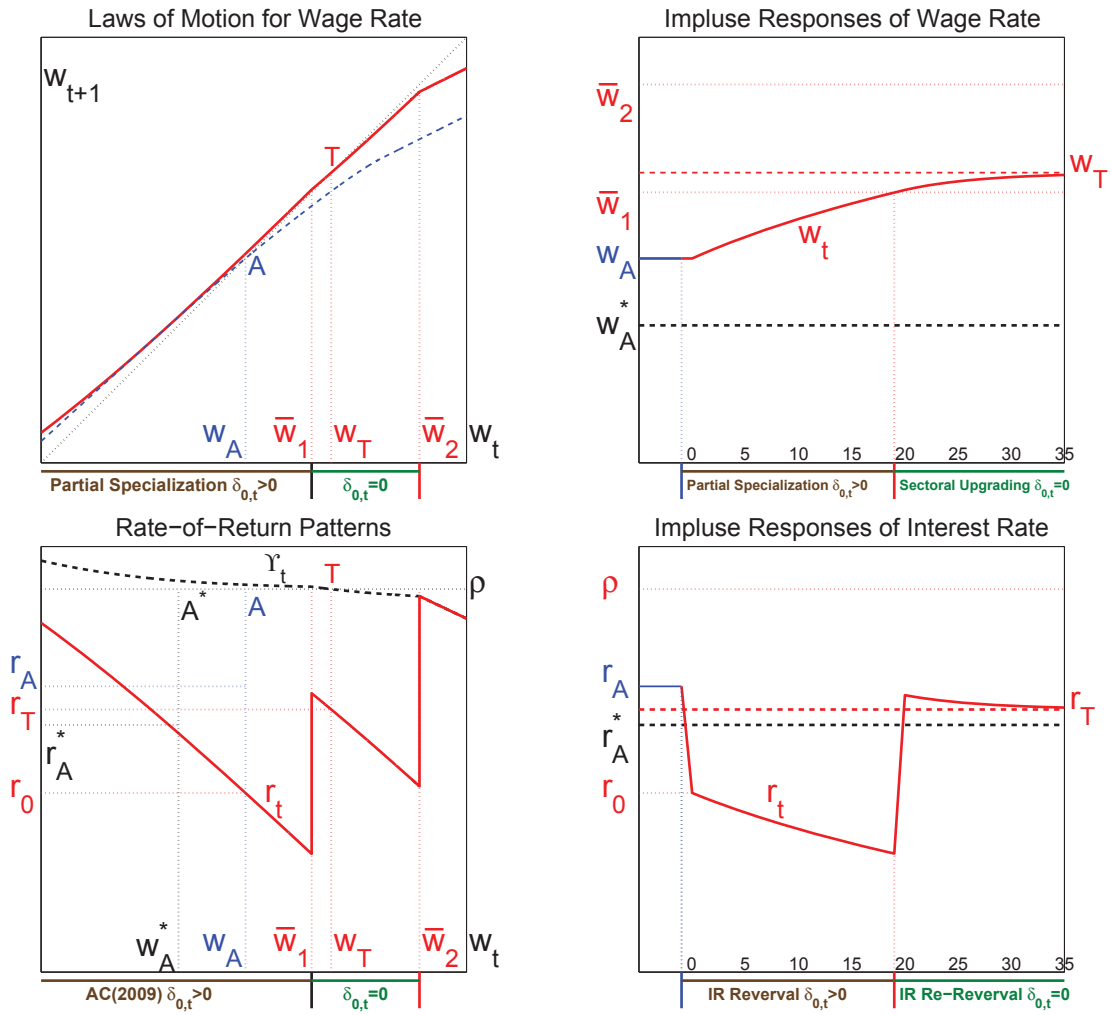

Figure 14: The Case of Moderate Heterogeneity in Financial Development

In the case of a moderate heterogeneity in financial development, figure 14 shows the dynamics of national income and the interest rate in North, where the notations follow those of figures 9 and 10. As described before, the interest rate has an inverse sawtooth wave pattern with respect to national income. Given $w_0 = w_A < \bar{w}_1$, North specializes partially towards sector 1 and 2 in period $t = 1$, which leads to the interest rate reversal in period 0, $r_0 < r_A^* < r_A$,

by the same logic as in subsection 3.2. Then, the rise in national income reduces the interest rate in North until period $t = 20$. In period $t = 21$, North fully offshores sector 0, leading to the interest rate re-reversal, $r_{21} > r_A^*$. Eventually, North converges to the new steady state with $w_T \in (\bar{w}_1, \bar{w}_2)$ and $r_T > r_A^*$.

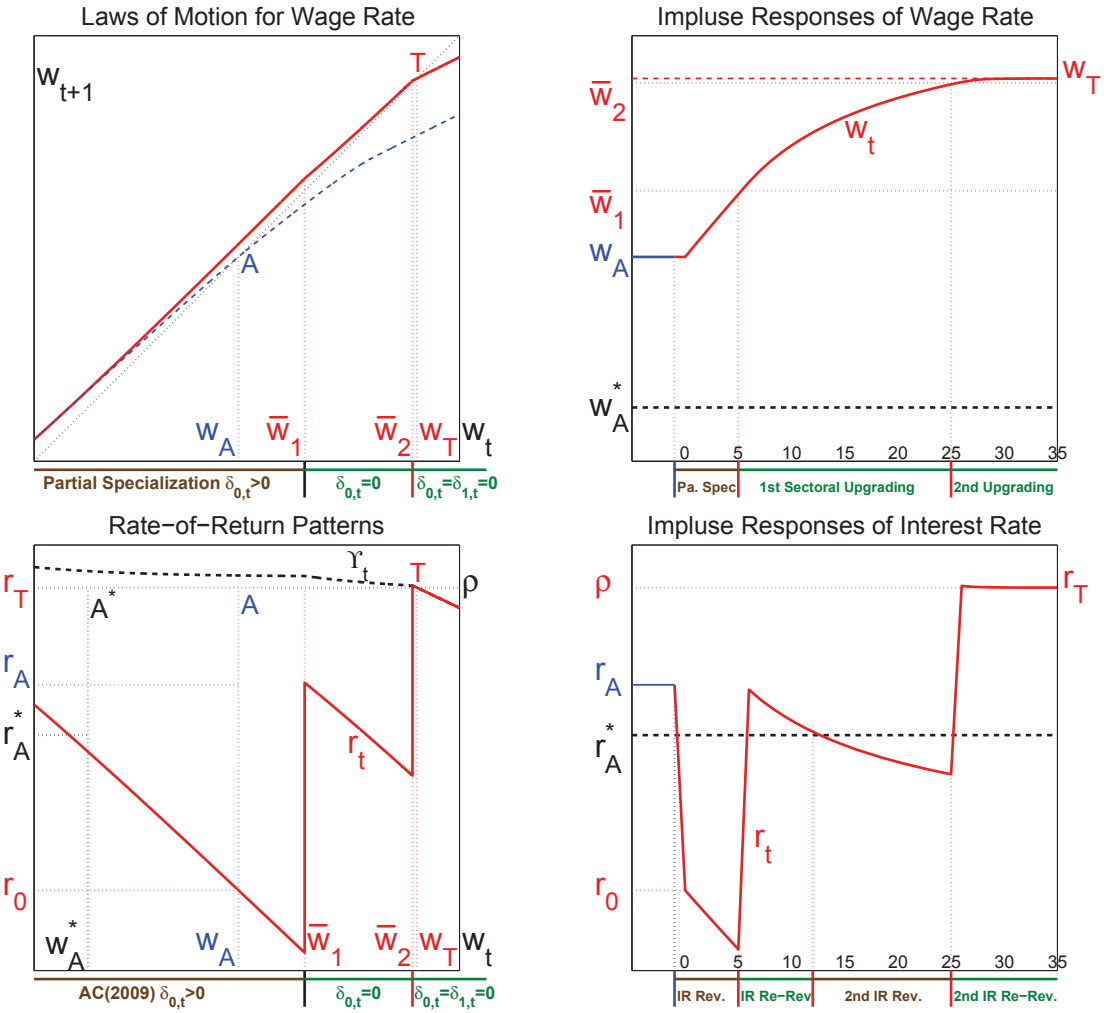

Figure 15: The Case of Large Heterogeneity in Financial Development

In the case of a large heterogeneity in financial development, figure 15 shows the dynamics of national income and the interest rate in North. In comparison with the previous case, the international relative price differentials are larger and so are trade flows and the gains from trade. Thus, North converges eventually to a new steady state with a higher national income, $w_T > \bar{w}_2$. During this process, the interest rate re-reversal happens twice, when North offshores sector 0 and 1 in period 6 and 26, respectively.

To sum up, the interest rate reversal and re-reversal can be recurrent if trade integration allows the more financially develop country to sequentially offshore the low-return sectors and upgrade to the high-return sectors over time.[31]

[31]This mechanism can be easily extended into a multi-sector setting. The technical analysis of a generalized, multi-sector model is available upon request.

As discussed in the beginning of this paper, "sectors" in my model can be interpreted as production stages or tasks. The empirical literature has documented extensively firms' heterogeneity in financial dependence in terms of size, industries, trade exposures, and etc.(Beck et al., 2008; Manova, 2013; Rajan and Zingales, 1998). My paper calls for the empirical investigation of heterogeneous financial dependence along production stages. Although it is almost impossible to decompose cross-stage financial dependence within a firm, one can study the firms that specialize at the different stages of supply chains.

Given cross-stage heterogeneity in financial dependence, cross-country heterogeneity in financial development can be the driving force behind global production network and supply-chain trade. As shown in figure 1, if supply-chain trade allows North to move partially away from the low-return production stages, the smile curve in North witnesses a *falling jaw*, which helps resolve the global imbalances. However, if supply-chain trade allows North to fully offshore the low-return activities, the smile curve in North witnesses a *missing jaw*, which may amplify the global imbalances. In the multi-sector setting, the sequential offshoring of low-return activities in North may have the non-monotone implications to international interest rate differentials and the global imbalances can be recurrent.

5 Final Remarks

This paper highlights the extensive margin of sectoral investment as a key channel through which trade integration affects domestic sectoral composition and the incentives for international financial flows in a world with heterogeneous financial development. Whether trade flows amplify or dampen the global imbalances depends on how far trade-driven sectoral shifts reshape the industrial structure in the more financially developed country, which fundamentally complements the findings of Antras and Caballero (2009). The logic of my model results can be applied to the case of supply-chain trade. In particular, supply-chain trade may amplify the global imbalances, if it allows the developed countries to offshore the low-return production activities and upgrade to the high-return activities.

For simplicity, I take the level of financial development as exogenously given and analyze the impacts of economic integration. In fact, trade and capital account liberalization may affect the incentives for financial intermediaries (Alessandria and Qian, 2005; Svaleryd and Vlachos, 2002; Tressel and Verdier, 2011) and the structures of financial markets from the political economy perspective (Braun and Raddatz, 2008). Besides, market-reform policies that improve domestic financial sectors and institutional structures are more fundamental to the CEECs' income convergence than simply reducing the barriers to trade or financial flows.[32] Taking into

[32]Since the 2004 EU enlargement, the adoption of EU laws and directives has improved financial sector quality in the CEECs by upgrading their legal, regulatory, and supervisory framework to the same standard as in the Western Europe, while the dominance of foreign banks in the CEECs' financial markets has also improved the quality of domestic banking sectors. As a result, the CEECs have witnessed rapid and massive capital inflows (von Hagen and Zhang, 2014).

account these factors, developing countries may use trade and capital account liberalization as a triggering device and combine them with other market-reform policies to promote financial development and productivity growth. These topics are left for future research.

References

Aghion, Philippe and Patrick Bolton. 1997. "A Theory of Trickle-Down Growth and Development." *Review of Economic Studies* 64 (2):151–172.

Alessandria, George and Jun Qian. 2005. "Endogenous Financial Intermediation and Real Effects of Capital Account Liberalization." *Journal of International Economics* 67 (1):97–128.

Amador, Joao and Sonia Cabral. 2016. "Global Value Chains: A Survey of Drivers And Measures." *Journal of Economic Surveys* 30 (2):278–301.

Antràs, Pol. 2015. *Global Production: Firms, Contracts, and Trade Structure*. Princeton University Press.

Antras, Pol and Ricardo Caballero. 2009. "Trade and Capital Flows: A Financial Frictions Perspective." *Journal of Political Economy* 117 (4):701–744.

Atkinson, Anthony, Thomas Piketty, and Emmanuel Saez. 2011. "Top Incomes in the Long Run of History." *Journal of Economic Literature* 49 (1):3–71.

Baldwin, Richard. 2013a. "Global Supply Chains: Why They Emerged, Why They Matter, and Where They Are Going." In *Global value chains in a changing world*, edited by Deborah K. Elms and Patrick Low. WTO, WTO Secretariat, 13–59.

———. 2013b. "Trade and Industrialization after Globalization's Second Unbundling: How Building and Joining a Supply Chain Are Different and Why It Matters." In *Globalization in an Age of Crisis: Multilateral Economic Cooperation in the Twenty-First Century*, edited by Robert Feenstra and Alan Taylor. National Bureau of Economic Research, NBER Inc., 165–212.

Baldwin, Richard, Rikard Forslid, and Tadashi Ito. 2015. "Unveiling the Evolving Sources of Value Added in Exports." Joint Research Program Series No. 161, IDE-JETRO.

Baldwin, Richard, Tadashi Ito, and Hitoshi Sato. 2014. "The Smile Curve: Evolving Sources of Value Aadded in Manufacturing." Joint Research Program Series, IDE-JETRO.

Baldwin, Richard and Javier Lopez-Gonzalez. 2015. "Supply-Chain Trade: A Portrait of Global Patterns and Several Testable Hypotheses." *The World Economy* 38 (11):1682–1721.

Banerjee, Abhijit V. and Benjamin Moll. 2010. "Why Does Misallocation Persist?" *American Economic Journal: Macroeconomics* 20 (1):189–206.

Banerjee, Abhijit V. and Andrew F. Newman. 1993. "Occupational Choice and the Process of Development." *Journal of Political Economy* 101 (2):274–298.

Barseghyan, Levon and Riccardo DiCecio. 2011. "Entry Costs, Industry Structure, and Cross-Country Income and TFP Differences." *Journal of Economic Theory* 146 (5):1828–1851.

Beck, Thorsten. 2002. "Financial Development and International Trade: Is There A Link?" *Journal of International Economics* 57 (1):107–131.

———. 2003. "Financial Dependence and International Trade." *Review of International Economics* 11 (2):296–316.

Beck, Thorsten, Asli Demirguc-Kunt, Luc Laeven, and Ross Levine. 2008. "Finance, Firm Size, and Growth." *Journal of Money, Credit and Banking* 40 (7):1379–1405.

Braun, Matias and Claudio Raddatz. 2008. "The Politics of Financial Development: Evidence from Trade Liberalization." *Journal of Finance* 63 (3):1469–1508.

Buera, Francisco J., Joseph Kaboski, and Yongseok Shin. 2011. "Finance and Development: A Tale of Two Sectors." *American Economic Review* 101 (5):1964–2002.

Caballero, Ricardo, Emmanuel Farhi, and Pierre-Olivier Gourinchas. 2008. "An Equilibrium Model of "Global Imbalances" and Low Interest Rates." *American Economic Review* 98 (1):358–93.

Chesnokova, Tatyana. 2007. "Immiserizing Deindustrialization: A Dynamic Trade Model with Credit Constraints." *Journal of International Economics* 73 (2):407–420.

Chor, Davin and Kalina Manova. 2012. "Off the Cliff and Back? Credit Conditions and International Trade during the Global Financial Crisis." *Journal of International Economics* 87 (1):117–133.

Corrado, Carol, Jonathan Haskel, Cecilia Jona-Lasinio, and Massimiliano Iommi. 2013. "Innovation and Intangible Investment in Europe, Japan, and the United States." *Oxford Review of Economic Policy* 29 (2):261–286.

Dauth, Wolfgang, Sebastian Findeisen, and Jens Suedekum. 2017. "Trade and Manufacturing Jobs in Germany." *American Economic Review Papers and Proceedings* 107:337–42.

Eaton, Jonathan, Samuel Kortum, Brent Neiman, and John Romalis. 2016. "Trade and the Global Recession." *American Economic Review* 106 (11):3401–38.

Erosa, Andres and Ana Hidalgo-Cabrillana. 2008. "On Finance as A Theory of TFP, Cross-Industry. Productivity Differences, and Economic Rents." *International Economic Review* 491 (2):437–473.

Gabaix, Xavier. 2009. "Power Laws in Economics and Finance." *Annual Review of Economics* 1 (1):255–294.

Galor, Oded and Joseph Zeira. 1993. "Income Distribution and Macroeconomics." *Review of Economic Studies* 60 (1):35–52.

Gereffi, Gary. 1999. "International Trade and Industrial Upgrading in the Apparel Commodity Chain." *Journal of International Economics* 48 (1):37–70.

Ghironi, Fabio and Marc J. Melitz. 2005. "International Trade and Macroeconomic Dynamics with Heterogeneous Firms." *Quarterly Journal of Economics* 120 (3):865–915.

Gourinchas, Pierre-Olivier and Helene Rey. 2014. "External Adjustment, Global Imbalances, Valuation Effects." In *Handbook of International Economics*, vol. 4, edited by Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff. Elsevier B.V., 585–645.

Grossman, Gene M. and Esteban Rossi-Hansberg. 2006. "The Rise of Offshoring: It's Not Wine for Cloth Anymore." In *Proceedings - Economic Policy Symposium - Jackson Hole*. Federal Reserve Bank of Kansas City, 59–102.

Hur, Jung, Manoj Raj, and Yohanes Riyanto. 2006. "Finance and Trade: A Cross-country Empirical Analysis on the Impact of Financial Development and Asset Tangibility on International Trade." *World Development* 34 (10):1728–1741.

Jin, Keyu. 2012. "Industrial Structure and Financial Capital Flows." *American Economic Review* 102 (5):2111–46.

Jones, Charles I. 2015. "Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality." *Journal of Economic Perspectives* 29 (1):29–46.

Ju, Jiandong, Kang Shi, and Shang-Jin Wei. 2014. "On the Connection between Intra-temporal and Intertemporal Trades." *Journal of International Economics.* 92(S1):S36.

Ju, Jiandong and Shang-Jin Wei. 2005. "Endowment versus Finance: A Wooden Barrel Theory of International Trade." IMF Working Papers No 05/123.

———. 2010. "Domestic Institutions and the Bypass Effect of Financial Globalization." *American Economic Journal: Economic Policy* 2 (4):173–204.

———. 2011. "When is Quality of Financial System a Source of Comparative Advantage?" *Journal of International Economics* 84 (2):178–187.

Kimura, Tatsuya. 2003. "The 'Smile Curve' Phenomenon in the Japanese Assembly-Type Manufacturing Industry." Fujitsu Research Institute Paper No. 167.

Kletzer, Kenneth and Pranab K. Bardhan. 1987. "Credit Markets and Patterns of International Trade." *Journal of Development Economics* 27 (1-2):57–70.

Koopman, Robert, Zhi Wang, and Shang-Jin Wei. 2014. "Tracing Value-Added and Double Counting in Gross Exports." *American Economic Review* 104 (2):459–94.

Kose, Ayhan, Eswar Prasad, Kenneth Rogoff, and Shang-Jin Wei. 2010. "Financial Globalization and Economic Policies." In *Handbook of Development Economics*, vol. 5. Elsevier B.V., 4283–4359.

Kuznets, Simon and Elizabeth Jenks. 1953. *Shares of Upper Income Groups in Income and Savings*. National Bureau of Economic Research, Inc.

Manova, Kalina. 2008. "Credit Constraints, Equity Market Liberalizations and International Trade." *Journal of International Economics* 76 (1):33–47.

———. 2013. "Credit Constraints, Heterogeneous Firms, and International Trade." *Review of Economic Studies* 80 (2):711–744.

Matsuyama, Kiminori. 2005. "Credit Market Imperfections and Patterns of International Trade and Capital Flows." *Journal of the European Economic Association* 3 (2-3):714–723.

Mendoza, Enrique G., Vincenzo Quadrini, and Jose-Victor Rios-Rull. 2009. "Financial Integration, Financial Deepness, and Global Imbalances." *Journal of Political Economy* 117 (3):371–416.

Midrigan, Virgiliu and Daniel Yi Xu. 2014. "Finance and Misallocation: Evidence from Plant-Level Data." *American Economic Review* 104 (2):422–58.

OECD. 2013. "Interconnected Economies Benefiting from Global Value Chains." Tech. rep., OECD Publishing, Paris.

Park, Albert, Gaurav Nayyar, and Patrick Low. 2013. "Supply Chain Perspectives and Issues: A Literature Review." World Trade Organization and Fung Global Institute, ISBN 978-92-870-3893-7.

Piketty, Thomas. 1997. "The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing." *Review of Economic Studies* 64 (2):173–89.

Piketty, Thomas and Emmanuel Saez. 2003. "Income Inequality in the United States, 1913-1998." *The Quarterly Journal of Economics* 118 (1):1–39.

Prasad, Eswar S., Raghuram Rajan, and Arvind Subramanian. 2006. "Patterns of International Capital Flows and Their Implications for Economic Development." In *Proceedings of the 2006 Jackson Hole Symposium*. Federal Reserve Bank of Kansas City, 119–158.

Rajan, Raghuram and Luigi Zingales. 1998. "Financial Dependence and Growth." *American Economic Review* 88 (3):559–86.

Svaleryd, Helena and Jonas Vlachos. 2002. "Markets for Risk and Openness to Trade: How are they Related?" *Journal of International Economics* 57 (2):369.

———. 2005. "Financial Markets, The Pattern of Industrial Specialization and Comparative Advantage: Evidence From OECD Countries." *European Economic Review* 49 (1):113–144.

Timmer, Marcel, Abdul Azeez Erumban, Bart Los, Robert Stehrer, and Gaaitzen de Vries. 2014. "Slicing Up Global Value Chains." *Journal of Economic Perspectives* 28 (2):99–118.

Tressel, Thierry and Thierry Verdier. 2011. "Financial Globalization and the Governance of Domestic Financial Intermediaries." *Journal of the European Economic Association* 9 (1):130–175.

von Hagen, Jürgen and Haiping Zhang. 2014. "Financial Development, International Capital Flows, and Aggregate Output." *Journal of Development Economics* 106:66–77.

Wynne, José. 2005. "Wealth as A Determinant of Comparative Advantage." *American Economic Review* 95 (1):226–254.

Ye, Ming, Bo Meng, and Shang-Jin Wei. 2015. "Measuring Smile Curves in Global Value Chains." Discussion Papers No.530, IDE-JETRO.

Zhang, Haiping. 2017. "Wealth Inequality and Financial Development: Revisiting the Symmetry Breaking Mechanism." *Economic Theory* 63 (4):997–1025.

Online Appendix

A Free-Trade Equilibrium

For $\theta \in (\underline{\theta}, 1)$ and $\lambda, \lambda^* \in [0, \tilde{\lambda}_A)$, four cases may arise under free trade, given $\chi^* = \chi^*_A$:

- Case **M**: **m**ultiple steady states arise, i.e., there are two stable steady states denoted by H and L, and one unstable steady state denoted by M. The steady states are ranked in terms of income level, $w_H > \bar{w} > w_M > w_L > w_A$.
- Case **UF1**: North converges to a **u**nique steady state where it **f**ully specializes in sector **1**;
- Case **UP1**: North converges to a **u**nique steady state where it **p**artially specializes in sector **1**;
- Case **UP0**: North converges to a **u**nique steady state where it **p**artially specializes in sector **0**;

Define some threshold values as follows,

$$\hat{Z} \equiv (1-\mathrm{g})^{\mathrm{g}-\eta}(1-\eta\mathrm{g})^{1-\mathrm{g}} \text{ and } \hat{\lambda}^*_T \equiv 1 - \left(\frac{Z}{\hat{Z}}\right)^{\rho(1-\theta)}, \text{ where } \mathrm{g} \equiv \frac{\theta}{\rho(1-\theta)} \tag{37}$$

$$\tilde{Z} \equiv \eta^{\frac{\mathrm{g}(\mathrm{g}-\eta)}{1-\eta}}(1-\eta+\eta^{\frac{\mathrm{g}}{1-\eta}+1})^{1-\mathrm{g}} \text{ and } \tilde{\lambda}^*_T \equiv 1 - \left(\frac{Z}{\tilde{Z}}\right)^{\rho(1-\theta)} \tag{38}$$

In the following, I show graphically the threshold conditions in the $\{\theta, Z, \lambda^*, \lambda\}$ space that define the four cases[33] as well as show the law of motion for wage in each case.

Under trade, multiple steady states arise if four conditions are satisfied simultaneously,

- $\theta \in (\underline{\theta}, \alpha)$, i.e., $\{\alpha, \theta\}$ in region M of the left panel of figure 16, and
- $Z < \hat{Z}$, i.e., $\{\theta, Z\}$ in region M of the right panel of figure 16, and
- $\lambda^* < \hat{\lambda}^*_T$ and $\lambda \in (\tilde{\lambda}_T, \hat{\lambda}_T)$, i.e., $\{\lambda^*, \lambda\}$ in region M of the left panel of figure 17.

In the upper-left panel of figure 18, the solid (dashed) curve shows the law of motion for wage under free trade (autarky) for case M.

In section 3, I focus on case UF1 which arises if four conditions are satisfied simultaneously, i.e., $\theta \in (\underline{\theta}, 1)$ and $Z < \tilde{Z}$, i.e., $\{\theta, Z\}$ in regions M and UF of the right panel of figure 16,

- given $\{\theta, Z\}$ in region UF of the right panel of figure 16, it is required that $\{\lambda^*, \lambda\}$ in region UF1 of the middle panel of figure 17, i.e., $\lambda^* < \tilde{\lambda}^*_T$ and $\lambda \in (\tilde{\lambda}_T, \tilde{\lambda}_A)$;

[33] The proof of proposition 2 provides the technical derivation for these threshold values.

- given $\{\theta, Z\}$ in region M of the right panel of figure 16, it is required that $\{\lambda^*, \lambda\}$ in region UF1 of the middle panel of figure 17, i.e, $\lambda^* \in (0, \hat{\lambda}_T^*)$ and $\lambda \in (\hat{\lambda}_T, \tilde{\lambda}_A)$, or alternatively, $\lambda^* \in (\hat{\lambda}_T^*, \tilde{\lambda}_T^*)$ and $\lambda \in (\tilde{\lambda}_T, \tilde{\lambda}_A)$.

In the upper-right panel of figure 18, the solid (dashed) curve shows the law of motion for wage under free trade (autarky) for case UF1.

Given $\theta \in (\underline{\theta}, 1)$, free trade induces North to move from the autarkic steady state to a **u**nique steady state where it specializes **p**artially in sector **1** (**0**) if parameters are in region region UP1 (UP0) of figure 17. The three panels in figure 17 correspond to $\{\theta, Z\}$ in the three regions of the right panel of figure 16.

In the two bottom panels of figure 18, the solid (dashed) curve shows the law of motion for wage under free trade (autarky) in case UP1 and UP0, respectively.

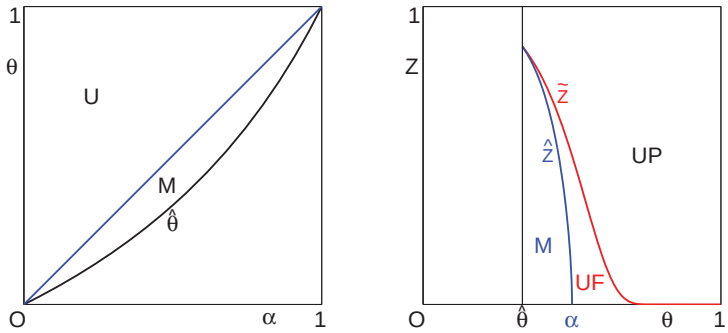

Figure 16: Threshold Values under Free Trade

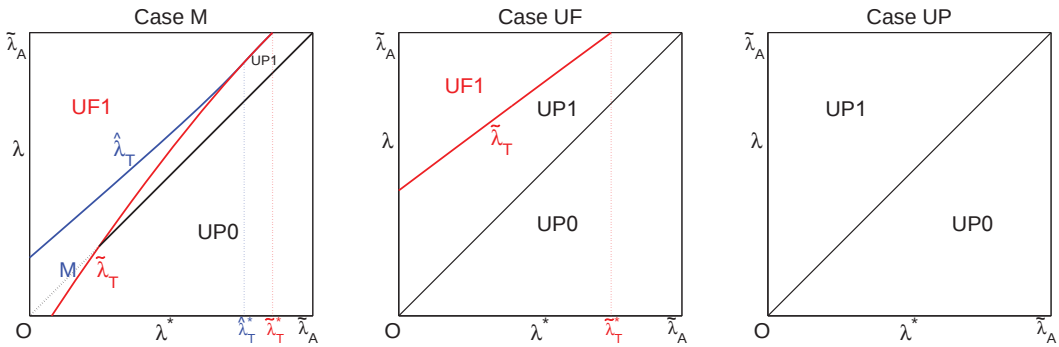

Figure 17: Threshold Values under Free Trade

B Proofs

Proof of Lemma 1

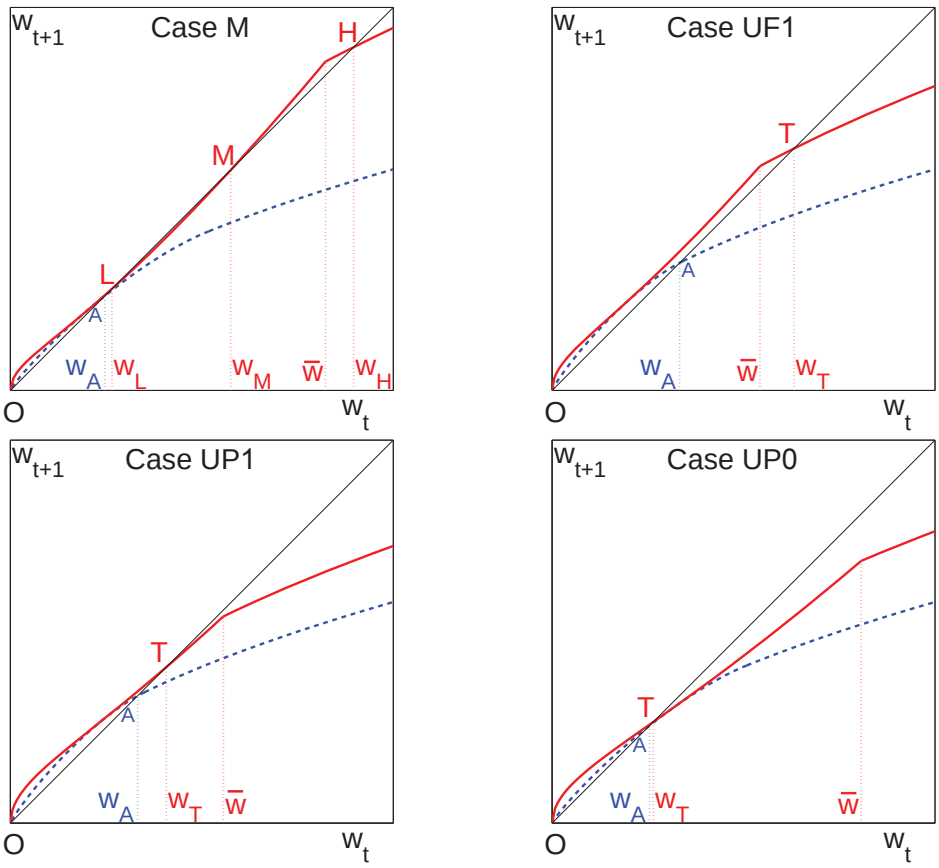

Figure 18: Laws of Motion for Wage under Free Trade: $\theta \in (\underline{\theta}, \alpha)$

Proof. **First, combine equations** (1) **and** (3)-(4) **to get equations** (5)-(6)**,**

$$\frac{q_{1,t+1}\delta_t w_t L}{\alpha} = p_{1,t+1}Y_{1,t+1} = \frac{w_{t+1}\zeta_{t+1}L}{1-\alpha}, \quad \frac{w_{t+1}}{q_{1,t+1}} = \frac{\delta_t w_t}{\rho\zeta_{t+1}},$$

$$\frac{q_{0,t+1}(1-\delta_t)w_t L}{\alpha} = p_{0,t+1}Y_{0,t+1} = \frac{w_{t+1}(1-\zeta_{t+1})L}{1-\alpha}, \quad \frac{w_{t+1}}{q_{0,t+1}} = \frac{(1-\delta_t)w_t}{\rho(1-\zeta_{t+1})},$$

$$\mu_{t+1} \equiv \frac{q_{0,t+1}}{q_{1,t+1}} = \frac{\frac{\delta_t}{1-\delta_t}}{\frac{\zeta_{t+1}}{1-\zeta_{t+1}}}, \quad \zeta_{t+1} = \frac{\delta_t}{(1-\delta_t)\mu_{t+1} + \delta_t} \tag{39}$$

$$Y_{1,t+1} = \left(\frac{\delta_t w_t L}{\alpha}\right)^{\alpha}\left(\frac{\zeta_{t+1}L}{1-\alpha}\right)^{1-\alpha} = \frac{p_{1,t+1}Y_{1,t+1}}{q_{1,t+1}^{\alpha}w_{t+1}^{1-\alpha}}, \tag{40}$$

$$Y_{0,t+1} = \left[\frac{(1-\delta_t)w_t L}{\alpha}\right]^{\alpha}\left[\frac{(1-\zeta_{t+1})L}{1-\alpha}\right]^{1-\alpha} = \frac{p_{0,t+1}Y_{0,t+1}}{q_{0,t+1}^{\alpha}w_{t+1}^{1-\alpha}}, \tag{41}$$

$$p_{1,t+1} = (q_{1,t+1})^{\alpha}w_{t+1}^{1-\alpha}, \quad p_{0,t+1} = (q_{0,t+1})^{\alpha}w_{t+1}^{1-\alpha}, \Rightarrow \chi_{t+1} = \mu_{t+1}^{\alpha}. \tag{42}$$

Second, derive the PPF in two cases.

- In the case of efficient sectoral investment, $\mu_{t+1} = 1$ and, according to equation (39), the sectoral shares of labor and capital equalize, $\zeta_{t+1} = \delta_t$. Let $\mathbb{Y}_{t+1} \equiv \left[\frac{w_t L}{\alpha}\right]^{\alpha}\left[\frac{L}{1-\alpha}\right]^{1-\alpha}$. Use equations (40)-(41) to get

$$Y_{1,t+1} = \zeta_{t+1}\mathbb{Y}_{t+1} \text{ and } Y_{0,t+1} = (1-\zeta_{t+1})\mathbb{Y}_{t+1}.$$

In this case, the PPF is specified by equation (7).

- In the case of inefficient sectoral investment, $\mu_{t+1} < 1$ and the borrowing constraints are binding. According to equation (12), sector 1's share of physical capital in period $t+1$, i.e., δ_t, is predetermined and depends on Y_t. Rewrite equations (40)-(41) as

$$\left(\frac{Y_{1,t+1}}{\delta_t^{\alpha}}\right)^{\frac{1}{1-\alpha}} = \zeta_{t+1}\mathbb{Y}_{t+1}^{\frac{1}{1-\alpha}} \text{ and } \left[\frac{Y_{0,t+1}}{(1-\delta_t)^{\alpha}}\right]^{\frac{1}{1-\alpha}} = (1-\zeta_{t+1})\mathbb{Y}_{t+1}^{\frac{1}{1-\alpha}}.$$

In this case, the PPF is specified by equation (17).

Third, derive the boundary condition for the two cases. According to equation (12), national income in period t determines sector 1's maximum share of physical capital in period $t+1$. Maintaining the efficient sectoral investment $\mu_{t+1} = 1$, sector 1's maximum possible share of domestic labor is $\zeta_{t+1} = \tilde{\delta}_t$, corresponding to the threshold output $\tilde{Y}_{1,t+1}$.

Fourth, derive the MRT from the gradient of the PPF. Combine (1) and (3)-(4),

$$\begin{aligned}
&\ln Y_{1,t+1} = \alpha \ln \delta_t + (1-\alpha)\ln \zeta_{t+1} + \ln \mathbb{Y}_{t+1},\\
&\ln Y_{0,t+1} = \alpha \ln(1-\delta_t) + (1-\alpha)\ln(1-\zeta_{t+1}) + \ln \mathbb{Y}_{t+1}\\
&\frac{\partial \ln Y_{1,t+1}}{\partial \ln \zeta_{t+1}} = \alpha\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1-\alpha),\ \frac{\partial \ln Y_{0,t+1}}{\partial \ln \zeta_{t+1}} = \frac{-\delta_t\alpha}{1-\delta_t}\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + \frac{-\zeta_{t+1}(1-\alpha)}{1-\zeta_{t+1}}\\
&MRT_{0,1} \equiv -\frac{\partial Y_{1,t+1}}{\partial Y_{0,t+1}} = -\frac{\partial \ln Y_{1,t+1}}{\partial \ln Y_{0,t+1}}\frac{Y_{1,t+1}}{Y_{0,t+1}} = \frac{\alpha\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1-\alpha)}{\alpha\frac{\delta_t}{1-\delta_t}\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} + (1-\alpha)\frac{\zeta_{t+1}}{1-\zeta_{t+1}}}\frac{Y_{1,t+1}}{Y_{0,t+1}}.
\end{aligned}$$

In the case of efficient sectoral investment, $\mu_{t+1} = 1$ and combine it with equation (39) to get $\zeta_{t+1} = \delta_t$ and $\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 1$. In the case of inefficient sectoral investment, $\mu_{t+1} < 1$ and, given the predetermined δ_t, $\frac{\partial \ln \delta_t}{\partial \ln \zeta_{t+1}} = 0$. Thus, the (absolute) gradient of the PPF is

$$MRT_{0,1} = \frac{1-\zeta_{t+1}}{\zeta_{t+1}}\frac{Y_{1,t+1}}{Y_{0,t+1}}. \tag{43}$$

□

Proof of Proposition 1

Proof. **First, derive the autarkic equilibrium.**

Use equation (2) to get the gradient of isoquant,

$$MRS_{0,1} \equiv -\frac{\partial V_{1,t+1}}{\partial V_{0,t+1}} = \frac{1-\eta}{\eta}\frac{V_{1,t+1}}{V_{0,t+1}}. \tag{44}$$

In the autarkic equilibrium, the PPF and the isoquant are tangent and the markets for sectoral outputs clear domestically. Combine equations (18), (43), and (44) to get

$$MRS_{0,1} = MRT_{0,1},\ \Rightarrow\ \frac{1-\eta}{\eta} = \frac{1-\zeta_{t+1}}{\zeta_{t+1}},\ \Rightarrow\ \zeta_{t+1} = \eta. \tag{45}$$

Under autarky, the sectoral share of domestic labor is equal to the sector's share in the production of final goods. Whether this also applies to the investment share depends on how far financial frictions distort the maximum share of domestic investment in sector 1.

$$\delta_t = \begin{cases} \tilde{\delta}_t < \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t < \eta; \\ \zeta_{t+1} = \eta, & \text{if } \tilde{\delta}_t \geq \eta. \end{cases} \tag{46}$$

Second, derive the law of motion for wage by combining equations (45)-(46) with (1)-(4) and (6) to get (22). In the case of efficient sectoral investment, $\mu_{t+1} = 1$ and the law of motion for wage degenerates into (21). In the case of inefficient sectoral investment, (12) specifies the fraction of domestic investment in sector 1 as a function of w_t. Combine (6) and (45) to get (23) which specifies the sectoral return wedge μ_{t+1} as a function of δ_t.

Third, derive the condition for the unique steady state, i.e., the steady state if exists is stable. For $w_t \geq \bar{w}_A$, the law of motion for wage in logarithm is linear with the slope less than unity, as shown by equation (21). If there exists a steady state with $w_A \geq \bar{w}_A$, it is stable. For $w_t < \bar{w}_A$, combine equations (12), (22), and (23)

$$\begin{aligned} \ln \delta_t &= \frac{1-\theta}{\theta}(\ln w_t - \ln \bar{w}), & \frac{\partial \ln \delta_t}{\partial \ln w_t} &= \frac{1-\theta}{\theta}, \\ \ln \mu_{t+1} &= \ln \delta_t - \ln(1-\delta_t) + \ln \frac{1-\eta}{\eta}, & \frac{\partial \ln \mu_{t+1}}{\partial \ln \delta_t} &= \frac{1}{1-\delta_t}, \\ \ln \Gamma_t &= \eta \ln \mu_{t+1} - \ln(1-\eta+\eta\mu_{t+1}), & \frac{\partial \ln \Gamma_t}{\partial \ln \mu_{t+1}} &= \eta(1-\mu_{t+1})(1-\delta_t). \end{aligned}$$

Combine them with equation (22) to get (24). The condition for the stable steady state is.

$$\frac{\partial w_{t+1}}{\partial w_t}|_{w_{t+1}=w_t} < 1, \;\; \Rightarrow \;\; \mu_A > 1 - \frac{\mathrm{g}}{\eta}, \;\; \text{where} \;\; \mathrm{g} \equiv \frac{\frac{1}{\alpha}-1}{\frac{1}{\theta}-1}. \tag{47}$$

Given $\mu_{t+1} \in (0, 1]$, a sufficient condition for inequality (47) to hold is

$$1 - \frac{\mathrm{g}}{\eta} \leq 0, \;\; \Rightarrow \;\; \theta \geq \underline{\theta} \equiv \frac{\eta\alpha}{\eta\alpha + (1-\alpha)}.$$

Consider first the case of $\theta \geq \underline{\theta}$, i.e., $\{\alpha, \theta\}$ in region U of the left panel of figure 4. Derive the condition under which the borrowing constraints are binding at the autarkic steady state, i.e., $\mu_A < 1$ or equivalently $\tilde{\delta}_A < \eta$. Combine $w_{t+1} = w_t = w_A$ with equations (12) and (22)-(23) to get $\mu_A < 1$ as a function of λ,

$$w_A = \left(\frac{1}{\rho}\frac{\mu_A^\eta}{1-\eta+\eta\mu_A}\right)^\rho = \delta_A^{\frac{\theta}{1-\theta}}\bar{w} = \left[\frac{\eta\mu_A}{1-\eta+\eta\mu_A}\right]^{\frac{\theta}{1-\theta}} \frac{\mathfrak{m}}{1-\theta}(1-\lambda)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \quad Z = \frac{\frac{1}{\rho}}{\left(\frac{\mathfrak{m}}{1-\theta}\eta^{\frac{\theta}{1-\theta}}\right)^{\frac{1}{\rho}}} = (1-\lambda)^{\frac{1}{\rho(1-\theta)}} \frac{(\mu_A)^{\mathrm{g}-\eta}}{(1-\eta+\eta\mu_A)^{\mathrm{g}-1}}, \tag{48}$$

$$\frac{\partial \ln \mu_A}{\partial \ln \lambda} = \frac{\frac{1}{\theta}\frac{\lambda}{1-\lambda}}{(1-\delta_A)[1-\frac{(1-\mu_A)\eta}{\mathrm{g}}]}. \tag{49}$$

Given $\theta > \underline{\theta}$, $\mathrm{g} > \eta$ and hence, $\frac{(1-\mu_A)\eta}{\mathrm{g}} < 1$ and hence, $\frac{\partial \ln \mu_A}{\partial \ln \lambda} > 0$. Let $\tilde{\lambda}_A$ denote the level of financial development that ensures $\tilde{\delta}_A = \eta$ and then $\mu_A = 1$. Combine them with equation (48),

$$\tilde{\lambda}_A \equiv 1 - Z^{\rho(1-\theta)}. \tag{50}$$

Finally, in the case of $\theta < \underline{\theta}$, i.e., $\{\alpha, \theta\}$ in region M of the left panel of figure 4, $\mathrm{g} < \eta$. Derive the threshold condition for multiple steady states, i.e., the existence of unstable steady state M. In the case of $\lambda = \hat{\lambda}_A$, the law of motion for wage is tangent with the 45° line at $w_M \in (0, \bar{w}_A)$,

$$\frac{\partial w_{t+1}}{\partial w_t}|_{w_M} = 1, \;\; \Rightarrow \;\; \mu_M = 1 - \frac{\mathrm{g}}{\eta}. \tag{51}$$

Combine equations (51) and (48) to get the threshold condition,

$$\hat{\lambda}_A = 1 - \left[Z \frac{(\frac{\eta - \mathrm{g}}{\eta})^{\eta - \mathrm{g}}}{(1-\mathrm{g})^{1-\mathrm{g}}} \right]^{\rho(1-\theta)}, \tag{52}$$

In the case of $\theta \in (0, \underline{\theta})$, figure 19 shows two threshold values, $\hat{\lambda}_A$ and $\tilde{\lambda}_A$, in the $\{\lambda, Z\}$ space. The solid curves in the three panels of figure 20 show the law of motion for wage, given $\{\lambda, Z\}$ in the three regions of figure 19, respectively, while the dashed curves show the laws of motion for wage in the benchmark case of $\lambda = 1$. The steady state properties in the three cases are summarized as follows.

- For $\{\lambda, Z\}$ in region N, the economy converges to the steady state with the income level at zero, $w_A = 0$. In this case, there does **n**ot exist a non-trivial steady state.
- For $\{\lambda, Z\}$ in region MB, there is an unstable steady state M and a stable steady state A with $0 < w_M < w_A < w_B$, and the borrowing constraints are **b**inding in steady state A.
- For $\{\lambda, Z\}$ in region MS, there is an unstable steady state M and a stable steady state A with $0 < w_M < w_A = w_B$, and the borrowing constraints are **s**lack in steady state A.

□

Proof of Lemma 2

Proof. Under autarky, the markets for sectoral outputs clear domestically, $V_{s,t} = Y_{s,t}$, implying $K_{1,t+1} > 0$ and $r_t = q_{0,t+1}$. According to (1)-(2), $w_{t+1}^{1-\alpha}(q_{0,t+1})^{\alpha(1-\eta)}(q_{1,t+1})^{\alpha\eta} = 1$. Combine them to get $w_{t+1} = \left(\frac{1}{r_t}\mu_{t+1}^{\eta}\right)^{\rho}$. Combine it with (22) to get (25).

For $\theta > \underline{\theta}$, there exists a unique steady state with $w_A = \left(\frac{\Gamma_A}{\rho}\right)^{\rho}$ where

$$\Gamma_A = \frac{\mu_A^{\eta}}{1 - \eta + \eta\mu_A} \;\; \text{and} \;\; \frac{\partial \ln \Gamma_A}{\partial \ln \mu_A} = \eta(1 - \delta_A) > 0, \;\; \text{given} \;\; \lambda \in [0, \tilde{\lambda}_A). \tag{53}$$

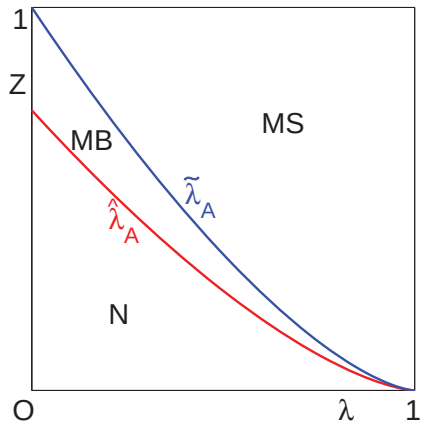

Figure 19: Steady-State Properties under Autarky

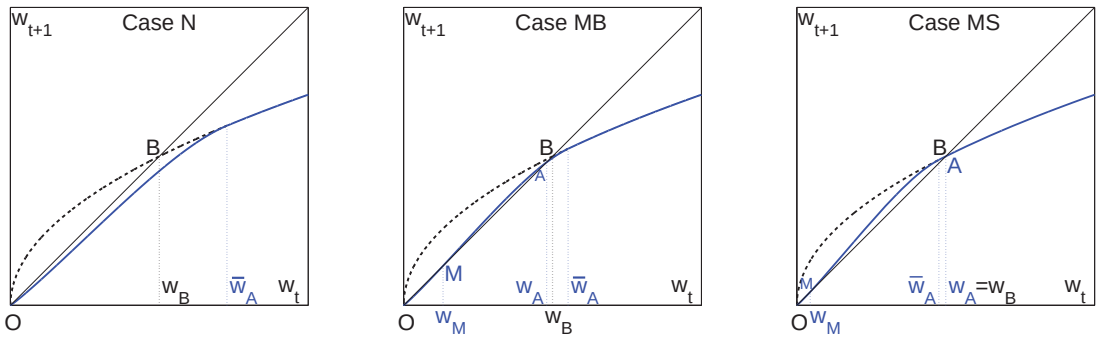

Figure 20: Laws of Motion for Wage under Autarky: $\theta < \underline{\theta}$

In the autarkic steady state, $w_{t+1} = w_t$ implies that $\Upsilon_A = \rho$ and $r_A = \rho[1-\eta+\eta\mu_A]$. According to equation (49), $\frac{\partial \mu_A}{\partial \lambda} > 0$ for $\theta > \underline{\theta}$. Thus, $\frac{\partial r_A}{\partial \lambda} > 0$, $\frac{\partial w_A}{\partial \lambda} > 0$, $\frac{\partial \ln \chi_A}{\partial \ln \lambda} > 0$. Given $\lambda^* < \lambda < \tilde{\lambda}_A$, it holds that $Y_A^* < Y_A$, $\chi_A^* < \chi_A < 1$, and $r_A^* < r_A < \rho$. □

Proof of Lemma 3

Proof. For $w_t < \bar{w}$, the maximum possible share of investment in sector 1 is less than unity. In this case, sector 0 is active $K_{0,t+1} > 0$ and the interest rate coupling gives $r_t = q_{0,t+1}$. Thus, the borrowing constraints are binding in North, $\mu_{t+1} = \frac{q_{0,t+1}}{q_{1,t+1}} = \frac{r_t}{q_{1,t+1}} = \mu_A^* < 1$ and $\delta_t = \tilde{\delta}_t < 1$ and $\zeta_{t+1} = \frac{1}{(\frac{1}{\delta_t}-1)\mu^*+1} \in (\delta_t, 1)$.

For $w_t \geq \bar{w}$, North offshores sector 0, $K_{0,t+1} = 0$, implying the interest rate decoupling from the rate of return in sector 0. The credit market competition leads to the interest rate coupling with the rate of return in sector 1. Combine them with equations (19), (1) and (3)-(6) to get (26). □

Proof of Lemma 4

Proof. Following the approach described in the proof of lemma 2 and taking into account equation (26), one can derive the interest rate as the piecewise function of national income characterized by (30)-(31). □

Proof of Proposition 2

Proof. For $\theta \in (\underline{\theta}, 1)$, there exists a unique, autarkic steady state. As the static gains from trade raise North's income, the law of motion for wage under free trade lies strictly above that under autarky. According to equations (28) and (29), the law of motion for wage in logarithm has a slope less than unity for $w_t > \bar{w}$. Thus, there must exist at least one stable steady state under free trade. In the following, I derive the threshold conditions for the steady-state properties under free trade.

First, derive the condition under which trade leads to multiple steady states in North. If there exists an unstable steady state M, it holds that $w_M \in (0, \bar{w})$ and

$$\frac{\partial w_{t+1}}{\partial w_t}|_{w_M} = \alpha + \alpha \frac{\frac{1-\theta}{\theta}}{\frac{\mu^*}{(1-\mu^*)\delta_M} + 1} > 1, \Rightarrow \frac{\alpha}{1-\alpha} - \frac{\theta}{1-\theta} > \frac{\theta}{1-\theta}\frac{\mu^*}{(1-\mu^*)\delta_M}. \tag{54}$$

For $\theta \geq \alpha$, condition (54) does not hold. In this case, the law of motion for wage in logarithm has the slope less than unity so that there exists a unique, stable steady state under free trade. In the following, I focus on the case of $\theta \in (\underline{\theta}, \alpha)$.

Combine condition (54) with $\delta_M = \left(\frac{w_M}{\bar{w}}\right)^{\frac{\theta}{1-\theta}} < 1$ to get an upper bound for μ^*,

$$\frac{\mu^*}{1-\mu^*}\frac{\mathrm{g}}{1-\mathrm{g}} < \delta_M < 1, \;\; \Rightarrow \;\; \mu^* \leq \hat{\mu}_T^* \equiv 1 - \mathrm{g}, \;\; \text{where} \;\; \mathrm{g} \equiv \frac{\frac{1}{\alpha} - 1}{\frac{1}{\theta} - 1} \in (\eta, 1). \tag{55}$$

According to equations (48)-(49), μ_A is an increasing function of λ. As South has the economic same structure as North, equations (48)-(49) also specify μ_A^* as an increasing function of $\lambda^* \in [0, \tilde{\lambda}_A)$. Combine (55) and (48) to get the upper bound for λ^*,

$$\lambda^* \leq \hat{\lambda}_T^* \equiv 1 - \left(\frac{Z}{\hat{Z}}\right)^{\rho(1-\theta)}, \;\; \text{where} \;\; \hat{Z} \equiv (1-\mathrm{g})^{\mathrm{g}-\eta}(1-\eta\mathrm{g})^{1-\mathrm{g}} < 1. \tag{56}$$

$Z \leq \hat{Z}$ ensures $\hat{\lambda}_T^* \geq 0$ and the existence of multiple steady states. $\hat{Z}$ is a function of θ, as shown by the curve between region M and UF in the right panel of figure 16.

Given $\{\theta, Z\}$ in region M of the right panel of figure 16, derive the condition in the $\{\lambda^*, \lambda\}$ space under which the law of motion for wage is tangent with the 45° line at steady state M,

$$\frac{\partial w_{t+1}}{\partial w_t}|_{w_M} = \alpha + \alpha \frac{\frac{1-\theta}{\theta}}{\frac{\mu^*}{(1-\mu^*)\delta_M} + 1} = 1, \;\; \Rightarrow \;\; \delta_M = \frac{\mathrm{g}}{1-\mathrm{g}}\frac{\mu^*}{(1-\mu^*)}. \tag{57}$$

Combine equations (17), (26) with (57) and get a threshold value of $\hat{\lambda}_T$ as a function of μ^*,

$$w_M = \left[\frac{(\mu^*)^\eta}{\rho}\left(1 + \frac{1-\mu^*}{\mu^*}\delta_M\right)\right]^\rho = \bar{w}\delta_M^{\frac{\theta}{1-\theta}} = (1-\hat{\lambda}_T)^{\frac{1}{1-\theta}}\frac{\mathfrak{m}}{1-\theta}\left[\frac{\mathrm{g}}{1-\mathrm{g}}\frac{\mu^*}{(1-\mu^*)}\right]^{\frac{\theta}{1-\theta}}$$

$$\hat{\lambda}_T = 1 - \left\{\frac{Z}{\left(\frac{\mathrm{g}}{(1-\mu^*)\eta}\right)^{\mathrm{g}}(\mu^*)^{\mathrm{g}-\eta}(1-\mathrm{g})^{1-\mathrm{g}}}\right\}^{\rho(1-\theta)}. \tag{58}$$

For each $\lambda^* \in [0, \hat{\lambda}_T^*]$, use equation (48) to solve for μ_A^* and then combine $\mu^* = \mu_A^*$ with (58) to solve for $\hat{\lambda}_T$. The curve between region M and UF1 in the left panel of figure 17 shows $\hat{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}_T^*]$.

Second, derive the condition under which North offshores sector 0 before reaching the stable steady state T. Given μ^*, use equation (26) to get,

$$w_T = \left[\frac{(\mu^*)^{\eta-1}}{\rho}\right]^{\rho} \geq \bar{w} = \frac{\mathfrak{m}(1-\lambda)^{\frac{1}{1-\theta}}}{1-\theta}, \Rightarrow \lambda \geq \tilde{\lambda}_T = 1 - \left[\frac{Z\eta^{\mathrm{g}}}{(\mu^*)^{1-\eta}}\right]^{\rho(1-\theta)}. \tag{59}$$

Given $\lambda \in [0, \tilde{\lambda}_A]$, combine equation (59) with (50) to get an upper bound for μ^*,

$$\tilde{\lambda}_T \leq \tilde{\lambda}_A \quad \Rightarrow \quad \mu^* < \tilde{\mu}_T^* = \eta^{\frac{\mathrm{g}}{(1-\eta)}}. \tag{60}$$

By the same logic as mentioned above, combine (60) and (48) to get an upper bound,

$$\lambda^* < \tilde{\lambda}_T^* \equiv 1 - \left(\frac{Z}{\tilde{Z}}\right)^{\rho(1-\theta)}, \quad \text{where} \quad \tilde{Z} \equiv \eta^{\frac{\mathrm{g}(\mathrm{g}-\eta)}{1-\eta}}(1-\eta+\eta^{\frac{\mathrm{g}}{1-\eta}+1})^{1-\mathrm{g}}.$$

$Z \leq \tilde{Z}$ ensures $\tilde{\lambda}_T^* \geq 0$. $\tilde{Z}$ is a function of θ and shown by the curve between region UF and UP in the right panel of figure 16.

Given $\{\theta, Z\}$ in region UF and M in the right panel of figure 16, for each $\lambda^* \in [0, \tilde{\lambda}_T^*]$, use equation (48) to solve for μ_A^* and then combine $\mu^* = \mu_A^*$ with (59) to solve for $\tilde{\lambda}_T$. For $\{\theta, Z\}$ in region UF of the right panel of figure 16, the curve between region UF1 and UP1 in the middle panel of figure 17 shows $\tilde{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}_T^*]$; for $\{\theta, Z\}$ in region M of the right panel of figure 16, the curve denoted by $\tilde{\lambda}_T$ in the middle panel of figure 17 shows $\tilde{\lambda}_T$ as a function of $\lambda^* \in [0, \hat{\lambda}_T^*]$. □

Proof of Proposition 2

Proof. Given the Cobb-Douglas production function at the aggregate level,

$$Y_{t+1} = \Pi_{s=0}^2 \left(\frac{V_{s,t+1}}{\eta_s}\right)^{\eta_s}, \; MRS_{x,s} = \frac{\eta_x}{\eta_s}\frac{V_{s,t+1}}{V_{x,t+1}}, \text{where } x, s \in \{0, ..., 2\}, \; s \neq x. \tag{61}$$

Following the proof of lemma 1, one can derive $MRT_{x,s} = \frac{\zeta_{x,t+1}}{\zeta_{s,t+1}}\frac{Y_{s,t+1}}{Y_{x,t+1}}$. Under autarky, the sectoral output markets clear domestically, $V_{s,t} = Y_{s,t}$. Combine it with $MRT_{x,s} = MRS_{x,s}$ to get the share of labor input in sector s at $\zeta_{s,t+1} = \eta_s$.

Given the Cobb-Douglas production function at the sectoral level,

$$\frac{q_{s,t+1}\delta_{s,t}w_tL}{\alpha} = p_{s,t+1}Y_{s,t+1} = \frac{w_{t+1}\zeta_{s,t+1}L}{1-\alpha}, \quad \frac{q_{s,t+1}\delta_{s,t}}{\zeta_{s,t+1}} = \frac{\rho w_{t+1}}{w_t} = \frac{q_{x,t+1}\delta_{x,t}}{\zeta_{x,t+1}}. \tag{62}$$

Thus, a sector's rate of return is inversely related to its capital-labor ratio.

First, solve for the sectoral investment shares $\delta_{s,t}$ in three cases. Case 1: if the borrowing constraints are binding in sector 1 and 2, the sectoral investment shares are specified by equations (33)-(35). Given $\zeta_{s,t+1} = \eta_s$, assumption 2 ensures $\frac{\delta_{1,t}}{\eta_1} > \frac{\delta_{2,t}}{\eta_2}$. Combine it with

equation (62) to get $q_{1,t+1} < q_{2,t+1}$. Thus, the agents with $\epsilon_j \geq \underline{\epsilon}_2$ borrow to the limit and invest in sector 2. Besides, $w_t < \bar{w}_{1,A}$ ensures $\frac{\delta_{0,t}}{\eta_0} > \frac{\delta_{1,t}}{\eta_1}$. Combine it with equation (62) to get $q_{0,t+1} < q_{1,t+1}$. Thus, the agents with $\epsilon_j \in [\underline{\epsilon}_1, \underline{\epsilon}_2)$ borrow to the limit and invest in sector 1. Thus, assumption 2 and $w_t < \bar{w}_{1,A}$ are the necessary and sufficient condition for this case to arise. **Case 2**: use the same logic to prove that $w_t \in [\bar{w}_{1,A}, \bar{w}_{2,A})$ ensure the binding (slack) borrowing constraints in sector 2 (1). In this case, $\delta_{2,t}$ is still specified by equation (33), while the capital-labor ratio equalizes in sector 0 and 1, i.e., $\delta_{s,t} = \frac{\eta_s}{\eta_0+\eta_1}(1-\delta_{2,t})$ for $s \in \{0,1\}$. **Case 3**: use the same logic to prove that $w_t > \bar{w}_{2,A}$ ensure the efficient sectoral investment shares, i.e., $\delta_{s,t} = \eta_s$.

Second, derive the conditions for the unique, autarkic steady state. Under autarky, $V_{s,t} = Y_{s,t}$. Combine it with $\zeta_{s,t+1} = \eta_s$ and equations (33)-(35) and (61) to get,

$$w_{t+1} = \left(\frac{w_t}{\rho}\Gamma_t\right)^{\alpha}, \text{where } \Gamma_t = \begin{cases} \Pi_{s=0}^{2}\left(\frac{\delta_{s,t}}{\eta_s}\right)^{\eta_s}, & \text{if } w_t < \bar{w}_1; \\ \left(\frac{1-\delta_{2,t}}{\eta_0+\eta_1}\right)^{\eta_0+\eta_1}\left(\frac{\delta_{2,t}}{\eta_2}\right)^{\eta_2}, & \text{if } w_t \in [\bar{w}_1, \bar{w}_2); \\ 1, & \text{if } w_t > \bar{w}_2. \end{cases} \tag{63}$$

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = \alpha + \alpha\frac{\partial \ln \Gamma_t}{\partial \ln w_t}, \text{where } \frac{\partial \ln \Gamma_t}{\partial \ln w_t} = \begin{cases} \frac{1-\theta}{\theta}\left(1-\frac{\eta_0}{\delta_{0,t}}\right), & \text{if } w_t < \bar{w}_1; \\ \frac{1-\theta}{\theta}\left(1-\frac{\eta_0+\eta_1}{1-\delta_{2,t}}\right), & \text{if } w_t \in [\bar{w}_1, \bar{w}_2); \\ 1, & \text{if } w_t > \bar{w}_2. \end{cases}$$

Given $\theta > \underline{\theta}_3 \equiv \frac{\alpha}{\alpha+\frac{1-\alpha}{\eta_1+\eta_2}}$, $\frac{\partial \ln w_{t+1}}{\partial \ln w_t} < 1$ holds strictly. As the slope of the law of motion for wage at any steady state is less than unity, the autarkic steady state is unique. In the following, I focus on the case of $\theta > \underline{\theta}_3$.

Finally, derive the threshold value $\tilde{\lambda}_{1,A}$ such that for $\lambda < \tilde{\lambda}_{1,A}$, the borrowing constraints are binding in sector 1 and 2 at the autarkic steady state. As the law of motion for wage is a piecewise function with two kinks at $w_t = \bar{w}_{1,A}$ and $w_t = \bar{w}_{2,A}$, one can solve $\tilde{\lambda}_{1,A}$ by making the first kink point a steady state, i.e., $w_A = \bar{w}_{1,A}$ and $\frac{\delta_{0,A}}{\eta_0} = \frac{\delta_{1,A}}{\eta_1} = \frac{1-\delta_{2,A}}{\eta_0+\eta_1}$. Let $\mathbb{D}_1 \equiv \frac{(1-\tilde{\lambda}_{1,A})^{\frac{1}{1-\theta}}}{1-\theta}$. Thus, $\bar{w}_2 = \mathfrak{m}_2\mathbb{D}_1$ and $\bar{w}_1 = \gamma\bar{w}_2$. Combine them with $w_A = \bar{w}_{1,A}$, the

definition of $\bar{w}_{1,A}$, and equations (33)-(35) and (63),

$$\delta_{2,A} = \left(\frac{w_A}{\bar{w}_2}\right)^{\frac{1-\theta}{\theta}} = \left(\frac{\bar{w}_{1,A}}{\mathbb{D}_1}\right)^{\frac{1-\theta}{\theta}} \mathfrak{m}_2^{-\frac{1-\theta}{\theta}},$$

$$\delta_{0,A} + \delta_{1,A} = 1 - \delta_{2,A} = \left(\frac{\bar{w}_{1,A}}{\mathbb{D}_1}\right)^{\frac{1-\theta}{\theta}} \mathfrak{m}_2^{-\frac{1-\theta}{\theta}} \frac{(\eta_0+\eta_1)}{\eta_1}\left(\gamma^{-\frac{1-\theta}{\theta}} - 1\right),$$

$$\rho w_A^{\frac{1}{\rho}} = \Gamma_A = \left(\frac{\delta_{0,A}+\delta_{1,A}}{\eta_0+\eta_1}\right)^{\eta_0+\eta_1}\left(\frac{\delta_{2,A}}{\eta_2}\right)^{\eta_2},$$

$$\rho\left(\frac{\bar{w}_{1,A}}{\mathbb{D}_1}\right)^{\frac{1}{\rho}} \mathbb{D}_1^{\frac{1}{\rho}} = \left(\frac{\bar{w}_{1,A}}{\mathbb{D}_1}\right)^{\frac{1-\theta}{\theta}} \mathfrak{m}_2^{-\frac{1-\theta}{\theta}} \left(\frac{\gamma^{-\frac{1-\theta}{\theta}}-1}{\eta_1}\right)^{1-\eta_2}\left(\frac{1}{\eta_2}\right)^{\eta_2}$$

$$\frac{\bar{w}_{1,A}}{\mathbb{D}_1} = \frac{\bar{w}_{1,A}\gamma\mathfrak{m}_2}{\bar{w}_1} = \frac{\gamma\mathfrak{m}_2}{\left[1+\frac{\eta_0}{\eta_1}\left(1-\gamma^{\frac{1-\theta}{\theta}}\right)\right]^{\frac{\theta}{1-\theta}}}$$

$$\mathbb{D}_1^{\frac{1}{\rho}} = \left(\frac{\bar{w}_{1,A}}{\mathbb{D}_1}\right)^{\frac{1-\theta}{\theta}-\frac{1}{\rho}} \frac{1}{\rho}\mathfrak{m}_2^{-\frac{1-\theta}{\theta}} \left(\frac{\gamma^{-\frac{1-\theta}{\theta}}-1}{\eta_1}\right)^{1-\eta_2}\left(\frac{1}{\eta_2}\right)^{\eta_2},$$

$$(1-\tilde{\lambda}_{1,A})^{\frac{1}{\rho(1-\theta)}} = \mathbb{K}Z_3, \;\; \Rightarrow \;\; \tilde{\lambda}_{1,A} = 1-(\mathbb{K}Z_3)^{\rho(1-\theta)},$$

where Z_3 and $\mathbb{K} > 1$ are defined in proposition 4.

One can derive another threshold value $\tilde{\lambda}_{2,A} \equiv 1 - Z_3^{\rho(1-\theta)}$ such that, for $\lambda = \tilde{\lambda}_{2,A}$, the autarkic steady state is at the other kink point $w_A = \bar{w}_{2,A}$. For $\lambda \in (\tilde{\lambda}_{1,A}, \tilde{\lambda}_{2,A})$, the borrowing constraints are binding only in sector 2 at the autarkic steady state; for $\lambda > \tilde{\lambda}_{2,A}$, the borrowing constraints are slack in all sectors at the autarkic steady state. □

Proof of Lemma 6

Proof. According to proposition 4, there is a unique steady state where $w_A < \bar{w}_{1,A}$ and the borrowing constraints are binding in sector 1 and 2. Let $\mathbb{H} \equiv \frac{\delta_{0,A}}{\delta_{2,A}} = \delta_{2,A}^{-1} - \gamma^{-\frac{1-\theta}{\theta}}$. At the autarkic steady state, combine $w_{t+1} = w_t = w_A$ with equations (33)-(35) and (63),

$$\rho w_A^{\frac{1}{\rho}} = \Gamma_A, \;\; \rho\delta_{2,A}^{\frac{\theta}{\rho(1-\theta)}}\bar{w}_2^{\frac{1}{\rho}} = \delta_{2,A}\left(\frac{\mathbb{H}}{\eta_0}\right)^{\eta_0}\left(\frac{\gamma^{-\frac{1-\theta}{\theta}}-1}{\eta_1}\right)^{\eta_1}\left(\frac{1}{\eta_2}\right)^{\eta_2},$$

$$\bar{w}_2^{\frac{1}{\rho}} = [\mathbb{H}+\gamma^{-\frac{1-\theta}{\theta}}]^{\frac{\theta}{\rho(1-\theta)}-1}\mathbb{H}^{\eta_0}\frac{(\gamma^{-\frac{1-\theta}{\theta}}-1)^{\eta_1}}{\rho\eta_0^{\eta_0}\eta_1^{\eta_1}\eta_2^{\eta_2}},$$

$$\frac{1}{\rho} = \left\{\eta_0 + \left[\frac{1}{\rho(\frac{1}{\theta}-1)}-1\right]\delta_{0,A}\right\}\frac{\partial\ln\mathbb{H}}{\partial\ln\bar{w}_2},$$

$$\text{for } \theta > \underline{\theta}_3, \;\; \eta_0 + \left[\frac{1}{\rho(\frac{1}{\theta}-1)}-1\right]\delta_{0,A} > \eta_0(1-\delta_{0,A}), \;\; \Rightarrow \;\; \frac{\partial\ln\mathbb{H}}{\partial\ln\bar{w}_2} > 0,$$

$$\frac{\partial\ln\bar{w}_2}{\partial\ln\lambda} = -\frac{\lambda}{(1-\theta)(1-\lambda)} < 0, \;\; \Rightarrow \;\; \frac{\partial\ln\mathbb{H}}{\partial\ln\lambda} = \frac{\partial\ln\mathbb{H}}{\partial\ln\bar{w}_2}\frac{\partial\ln\bar{w}_2}{\partial\ln\lambda} < 0. \tag{64}$$

At the autarkic steady state, combine $\zeta_{s,t+1} = \eta_s$ with equations (33)-(35) and (62),

$$\mu_{0,A} \equiv \frac{q_{0,A}}{q_{2,A}} = \frac{\frac{\delta_{2,A}}{\eta_2}}{\frac{\delta_{0,A}}{\eta_0}} = \frac{\eta_0}{\eta_2 \mathbb{H}}, \qquad \frac{\partial \ln \mu_{0,A}}{\partial \ln \lambda} = -\frac{\partial \ln \mathbb{H}}{\partial \ln \lambda} > 0,$$

$$\mu_{1,A} \equiv \frac{q_{1,A}}{q_{2,A}} = \frac{\frac{\delta_{2,A}}{\eta_2}}{\frac{\delta_{1,A}}{\eta_1}} = \frac{\eta_1}{\eta_2(\gamma^{-\frac{1-\theta}{\theta}} - 1)}, \qquad \frac{\partial \ln \mu_{1,A}}{\partial \ln \lambda} = 0.$$

Thus, given $\lambda^* < \lambda < \tilde{\lambda}_{1,A}$, $\mu^*_{0,A} < \mu_{0,A}$ and $\mu^*_{1,A} = \mu_{1,A}$ hold. According to equation (36), the ascending sectoral rate of return in each country implies,

$$\mu^*_{0,A} < \mu_{0,A} < \mu^*_{1,A} = \mu_{1,A} < \mu^*_{2,A} = \mu_{2,A} = 1. \tag{65}$$

For $s \in \{0, 1, 2\}$, use equation (62) to get

$$q_{s,t+1} = \alpha p_{s,t+1} \left(\frac{\delta_{s,t}}{\zeta_{s,t+1}} \right)^{\alpha-1} \mathbb{Y}_{t+1}, \quad \mu_{s,t+1} \equiv \frac{q_{s,t+1}}{q_{2,t+1}} = \frac{\frac{\delta_{2,t}}{\zeta_{2,t+1}}}{\frac{\delta_{s,t}}{\zeta_{s,t+1}}}, \tag{66}$$

$$\mu_{s,t+1} = \chi_{s,t+1} \left(\frac{\frac{\delta_{s,t}}{\zeta_{s,t+1}}}{\frac{\delta_{2,t}}{\zeta_{2,t+1}}} \right)^{\alpha-1} = \chi_{s,t+1} \mu_{s,t+1}^{1-\alpha}, \quad \Rightarrow \quad \chi_{s,t+1} = \mu_{s,t+1}^{\alpha}, \tag{67}$$

which holds under autarky as well as under free trade. Combine it with inequality (65) to get the cross-country patterns of the relative sectoral prices,

$$\chi^*_{0,A} < \chi_{0,A} < \chi^*_{1,A} = \chi_{1,A} < \chi^*_{2,A} = \chi_{2,A} = 1.$$

Combine $r_t = q_{0,t+1}$ with equations (62) and (64) to get the steady-state interest rate,

$$r_A = \frac{\rho}{\frac{\delta_{0,A}}{\eta_0}} = \rho \eta_0 \left(1 + \frac{1}{\mathbb{H} \gamma^{\frac{1-\theta}{\theta}}} \right), \ \frac{\partial r_A}{\partial \mathbb{H}} < 0, \ \Rightarrow \ \frac{\partial r_A}{\partial \lambda} = \frac{\partial r_A}{\partial \mathbb{H}} \frac{\partial \mathbb{H}}{\partial \lambda} > 0, \ \Rightarrow \ r^*_A < r_A.$$

□

Proof of Proposition 5

Proof. Under free trade, the relative sectoral prices in North are aligned to the world levels, $\chi_{s,t} = \chi^*_s$, and so are the relative sectoral rental prices of capital, $\mu_{s,t} = \chi_{s,t}^{\frac{1}{\alpha}} = (\chi^*_s)^{\frac{1}{\alpha}} = \mu^*_s$, according to equation (67).

For expositional convenience, define $\bar{w}_0 \equiv 0$ and $\bar{w}_3 \equiv \infty$. As shown in subsection 4.3.2, trade may allow North to offshore the low-return sectors. Let $\mathbb{a} \equiv \min\{z : \delta_{z,t} > 0\}$ denote the lowest sector index among all **active** sectors in North. For $w_t \in (\bar{w}_{\mathbb{a}}, \bar{w}_{\mathbb{a}+1})$, North offshores sector $u < \mathbb{a}$ and specializes in sector $s \geq \mathbb{a}$.

Combine $\mu_{s,t+1} = \mu^*_s$ with equation (66) and the labor market clearing condition,

$$\zeta_{s,t+1} = \frac{\mu^*_s}{\frac{\delta_{2,t}}{\zeta_{2,t+1}}} \delta_{s,t}, \quad \sum_{v=\mathbb{a}}^{2} \zeta_{v,t+1} = 1, \quad \frac{\delta_{2,t}}{\zeta_{2,t+1}} = \sum_{v=\mathbb{a}}^{2} \mu^*_v \delta_{v,t}, \quad \frac{\delta_{s,t}}{\zeta_{s,t+1}} = \frac{\sum_{v=\mathbb{a}}^{2} \mu^*_v \delta_{v,t}}{\mu^*_s}$$

Let $\Gamma_t \equiv \frac{\sum_{v=\mathbb{a}}^{2} \mu_v^* \delta_{v,t}}{\Pi_{v=0}^{2} (\mu_v^*)^{\eta_v}}$. Derive the law of motion for wage under free from the aggregate production function and the national income-expenditure identity,

$$Y_{t+1} = \Pi_{v=0}^{2} \left(\frac{V_{v,t+1}}{\eta_v} \right)^{\eta_v}, \; p_{s,t+1} \frac{V_{s,t+1}}{\eta_s} = Y_{t+1}, \; \Pi_{v=0}^{2} p_{v,t+1}^{\eta_v} = 1, \; p_{s,t+1} = p_{2,t+1} \chi_s^*,$$

$$p_{2,t+1} = \frac{1}{\Pi_{v=0}^{2} (\chi_v^*)^{\eta_v}}, \;\; p_{s,t+1} = \frac{\chi_s^*}{\Pi_{v=0}^{2} (\chi_v^*)^{\eta_v}} = \left[\frac{\mu_s^*}{\Pi_{v=0}^{2} (\mu_v^*)^{\eta_v}} \right]^{\alpha}$$

$$p_{s,t+1} Y_{s,t+1} = \left[\frac{\mu_s^*}{\Pi_{v=0}^{2} (\mu_v^*)^{\eta_v}} \right]^{\alpha} \left(\frac{\delta_{s,t}}{\zeta_{s,t+1}} \right)^{\alpha} \zeta_{s,t+1} \mathbb{Y}_{t+1} = \zeta_{s,t+1} \Gamma_t^{\alpha} \mathbb{Y}_{t+1},$$

$$w_{t+1} = \frac{(1-\alpha) Y_{t+1}}{L} = \frac{(1-\alpha) \sum_{v=\mathbb{a}}^{2} p_{v,t+1} Y_{v,t+1}}{L} = \left(\frac{w_t}{\rho} \Gamma_t \right)^{\alpha}.$$

According to lemma 6, all sectors are active in the autarkic steady state, $\mathbb{a} = 0$, with $\mu_{0,A}^* < \mu_{0,A} < \mu_{1,A} = \mu_{1,A}^* < 1$. Given $w_0 = w_A$, the static gains from trade are measured by the response of the aggregate efficiency index.

$$\frac{\partial \ln w_1}{\partial \ln \chi_1} = \frac{\partial \ln \Gamma_1}{\partial \ln \mu_1} = \frac{\mu_0^* - \mu_{0,A}}{\frac{\mu_{0,A}^*}{\eta_1 + \eta_2} + \frac{\mu_{0,A}}{1-(\eta_1+\eta_2)}},$$

which is equivalent to equation (27) in the two-sector setting. Given $\lambda^* < \lambda < \tilde{\lambda}_{1,A}$, the lower λ^*, the larger the heterogeneity in financial development $\lambda - \lambda^*$, the larger the international relative price differential $\chi_{0,A} - \chi_0^*$ and the international relative rate-of-return differential in sector 0, $\mu_{0,A} - \mu_0^*$, the larger the trade flows and the static gains.

From period $t = 1$ on, the rise in national income triggers the sectoral investment reallocation towards the high-MIR, high-return sectors along the extensive margin, which improves aggregate efficiency indicator Γ_t and raises national income in period $t + 1$.

$$\frac{\partial \ln w_{t+1}}{\partial \ln w_t} = 1 - (1-\alpha) + \alpha \sum_{s=0}^{2} \frac{\partial \ln w_{t+1}}{\partial \ln \delta_{s,t}} \frac{\partial \ln \delta_{s,t}}{\partial \ln w_t} = \alpha + \alpha \frac{\frac{1-\theta}{\theta}}{1 + \frac{1}{(\frac{\mu_1^*}{\mu_0^*} - 1)\delta_{1,t} + (\frac{1}{\mu_0^*} - 1)\delta_{2,t}}},$$

which is similar as equation (28) in the two sector-setting. The dynamics of national income are driven by the competition of the decreasing MRK effect and the investment reallocation effect in the case of $w_t < \bar{w}_1$. Given λ, the lower the λ^*, the lower the μ_0^*, the stronger the investment reallocation effect, the larger the dynamic gains from trade. If $\lambda - \lambda^*$ is sufficiently large, the static and the dynamic gains from trade can be so strong that North fully offshores the low-MIR, low-return sector(s). □